Evolution of 3D unsteady water wave modulations

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A number of attempts have been reported in the literature where freak waves are produced by nonlinear self-modulation of a two-dimensional slowly modulated wave train. To do so, both solutions based on the nonlinear Schrödinger equation (NLS) or its modifications [6], and numerical models solving fully nonlinear free surface flows have been proposed ([3],[4]). In these studies three-dimensional (3D) effects were usually neglected, either because it was not possible to generalize the methods of solution to three dimensions, or because the computational effort in a 3D model was still too high.

Since many extreme (freak) waves are expected (and have been observed) to be 3D, however, 3D modulational instabilities cannot be neglected when describing the steepest waves. Indeed, as theoretically predicted by McLean [2], and confirmed in the experimental observations by Su et al. [1], a type of instability exists (called type II) which is predominantly three-dimensional, in contrast with the Benjamin-Feir instability (type I) which is two-dimensional in nature.

The goal of this work is to study extreme waves produced by the evolution of 3D wave trains subjected to both longitudinal and lateral modulations. The computationally efficient Higher Order Spectral (HOS) method [4] is used in the present computations, assuming doubly periodic boundary conditions in the computational domain. A transversal modulation was superimposed to a longitudinal one. Modulations of this type are characterized by the initial steepness of the wave train (ak), and two characteristic wavelenghts, for the longitudinal and transversal modulations, respectively. When the transversal modulation wavelength is sufficiently large, one can observe the growth of the lateral modulation through the absorbtion of part of the longitudinal wave energy. The steeper the wavetrain, the more important this effect. The modulational wavelengths are the fundamental parameters governing wave train evolution.

The modulation growth should be limited by wave breaking, which cannot be described by this HOS model. Breaking will not happen uniformly along a wave crest, and a 3D self-focussed breaking wave is expected to appear at some stage of the modulation. In a second part of this work, breaking is studied using an accurate 3D-BEM fully nonlinear potential flow model, with an Eulerian-Lagrangian flow representation, recently developed by Grilli et al. [5]. Very large, possibly breaking (i.e., overturning), 3D transient waves can be modeled in this 3D-BEM model, by using the HOS method to compute the first stages of growth of wave modulations (the longer ones, on the order of 100 wave periods) as initial condition for the model. Typical 3D-HOS results can be found below. 3D-BEM results will be presented during the workshop.

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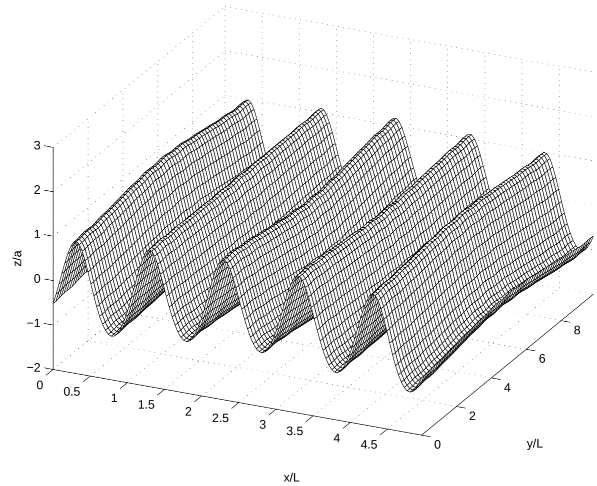

Figure 1: Initial condition for a 3D wave train modulated in both the longitudinal and the transversal direction

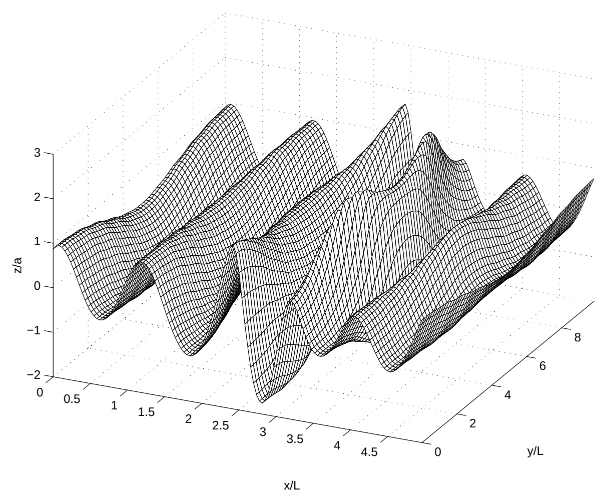

Figure 2: Evolution of the wave train in fig. 1 after $t/T = 90$

References

[1] Su, M.Y., Bergin, M., Marler, P., Myrick, R. Experiments on nonlinear instabilities and evolution of steep gravity-wave trains. *J. Fluid Mech.* 1982; **124**: 45-72.

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[4] Dommermuth, D.G., Yue, D.K.P., A higher-order spectral method for the study of non linear gravity waves. *J. Fluid Mech.* 1987; **184**: 267-288.

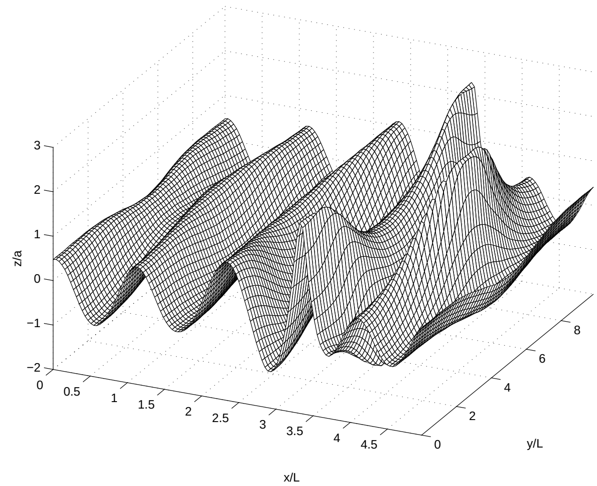

Figure 3: Evolution of the wave train in fig. 1 after $t/T = 91$

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