

Modelling a "rogue wave" - speculations or a realistic possibility?

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It seems that freak- or rogue waves exist even in the open ocean far away from strong current gradients (Kjeldsen (1984), Sand et.al. (1990), Skourup et.al.(1996)). Skourup et.al.(1996) analyzed 89 storms over a 12 year period at the Gorm field in the central North Sea. They used the following criteria to select candidates for their rogue wave collection: Single waves with crest heights, $a_c > 1.1H_s$,¹ or wave heights larger than $2H_s$, where H_s is the significant wave height of the surrounding 20 min. wave record. They found 446 events satisfying the crest criterion. Although years of wave data from numerous buoys have been analyzed, the number of freak wave events recorded are still modest. The chances that such a wave hits a buoy is even lower than one previously expected, as pointed out by Magnusson et.al. (1999).

The interest in these waves is not only because of our rather limited knowledge of their statistical probability of occurrence. We need to know more about their dynamics, what they look like, how long they last and so on. Boccotti (1981) have investigated the expected configuration in space and time surrounding extremely high crests in a random Gaussian wavefield. The most likely configuration is found to resemble the auto-correlation function for the wavefield. Tests of this result so far seems rather inconclusive (Phillips et.al. (1993)).

What about the physics behind rogue waves? We know a number of mechanisms that will produce large waves from moderately small ones by focusing the energy: Basically there are three types of effects:

Spatial focussing. In deep water this is due to current refraction. Far offshore on the open ocean with only very small current velocities (less than 20cm/s say) these effects would seem to be negligible. White and Fornberg (1998) have pointed out, however, that even small random current fluctuations with rms values of the order 10cm/s can give focussing provided their scale is sufficiently large (of the order of 10km).

Temporal-spatial focusing. This is the result of dispersion and a chirped spatial distribution of frequency. In wave tanks the mechanism is used for producing short groups of large waves at a given position. It is done by producing a long and chirped wavegroup (with steadily decreasing frequency) by the wavemaker. With proper design of the frequency chirp, dispersion brings this group to contract to a few wavelengths at a given position. This type of focussing has been suggested by Pelinovsky and Kahrif (2000) as a possible explanation for freak waves. They show (using the KdV equation for shallow water waves) that if a given chirped wavetrain produces strong focusing in the absence of other waves, it will still do so (although somewhat weaker) when a random wavefield is added. If the amplitude of the deterministic chirped wavetrain is below the rms value of the random waves, it will remain "invisible" until it focuses.

Nonlinear focusing. The so-called Benjamin Feir (BF) instability of regular wavetrains is well-known. Henderson, Peregrine and Dold (1999) have investigated what they call steep wave events (SWE) by simulating the evolution of a periodically perturbed regular wavetrain. Due to the BF instability the wavetrain breaks up into periodic groups. Within each group a further focusing takes place producing a very large wave having a steepness roughly 3 times the initial steepness of the wavetrain. They suggest (see also Dysthe and Trulsen (1999)) that the SWE they see can be roughly modelled by the so-called breather solutions of the NLS equation.

It seems that all these mechanisms need some special preparation or coherence to work: For the spatial focussing to give sharp caustics, a regular incoming wave is needed. When the incoming waves are irregular with a distribution of frequencies and directions, the caustics are smeared out and disappear, leaving only more diffuse variations in the wave energy density (Trulsen et.al. (1990)).

For the temporal-spatial focussing to work a spatial ordering of frequencies in a chirped wavetrain is needed.

The nonlinear focussing as described by Henderson et. al. need a very narrow frequency band for the BF instability to work (Alber (1978)).

¹The probability $P(a_c > 1.1H_s)$ for such an events to happen according to the Rayleigh distribution is roughly $\simeq 6 \cdot 10^{-5}$. Ratios of $a_c/H_s > 2$ has been reported (Sand et.al. (1990))

Does this leave us with the old idea that the rogue waves are simple (and unlikely) constructive interference phenomena that can be explained by linear- or slightly (second order) nonlinear theory? This seems to be a rather popular assumption, and serves as a basis for the statistical estimates.

Another possibility, however, is that weak (third order) nonlinear wave interactions may play a role. Although these interactions are slow they are known to produce large waves under special conditions. The correlation they introduces between the interacting waves may change the probability of constructive interference.

I think it is fair to say that nobody knows the answer to these questions yet. To test this latter idea, a project funded by The Norwegian Research Council is presently starting up. The idea is to simulate a piece of the ocean surface of dimensions approximately 100x100 wavelengths. Starting with a wavefield based on a suitably truncated empirical spectrum (like JONSWAP) we will use the numerical model described by Trulsen et.al. (2000) to follow the evolution of the wavefield. The probability of seeing a freak wave event in a simulation is estimated to be more than 10^4 times higher than for a corresponding point measurement (buoy) over the same period of time.

References

- [1] I.E.Alber : The effect of randomness on the stability of two-dimensional surface wave trains. Proc.Roy.Soc. A 363 (1978) pp. 528..
- [2] P.Boccotti : On the highest waves in a stationary Gaussian process. Atti.Acc.Lig. 38 (1981) pp. 45-73.
- [3] K.B.Dysthe and K.Trulsen : Note on breather type solutions of the NLS as model for freak waves. Phys.Scripta T82 (1999) pp. 48-52.
- [4] K.L.Henderson, D.H.Peregrine and J.W.Dold : Unsteady water wave modulations: fully nonlinear solutions and comparison with the NLS equation. Wave Motion 29 (1999) pp. 341-361.
- [5] S.P.Kjeldsen : Dangerous wave groups. Norwegian Maritime Reseach. 12 (1984) pp. 453-473.
- [6] A.K.Magnusson, M.A.Donelan and W.M.Drennan : On estimating extremes in an evolving wavefield. Coastal Engineering 36 (1999) pp. 147-163.
- [7] O.M.Phillips, D.Gu and M.Donelan : Expected structure of extreme waves in a Gaussian sea. Part I: Theory and SWADE buoy measurements. J.Phys.Oceanogr. 23 (1993) pp. 992-1000.
- [8] S.E.Sand, N.E.O.Hansen, P.Klinting, O.T.Gudmestad and M.J.Sterndorff : Freak wave kinematics.
- [9] J.Skourup, K.K.Andreasen and N.E.O.Hansen : Non-Gaussian extreme waves in the central North Sea. 1996 OMAE, Vol I, part A, Offshore technology. ASME 1996.
- [10] G.N.Trulsen, K.B.Dysthe and J.Trulsen : Evolution of a gravity wave spectrum through a current gradient. J.Geophys.Res. 95 (1990) pp. 22141-22151.
- [11] K.Trulsen, I.Kliakhandler, K.B.Dysthe, and M.G.Velarde : On weakly nonlinear modulation of waves on deep water. Phys.Fluids 12 (2000) pp. 2432-2437.
- [12] B.S.White and B.Fornberg : On the chance of freak waves at sea. J.Fluid Mech. 355 (1998) pp.113-138.