Statistical modelling of asymmetric waves

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In a Gaussian wave model the extreme crest height distribution can be calculated, both exactly from the true energy spectrum or approximately from the Rayleigh approximation. Ocean wave exhibit more or less non-Gaussian distributions for the immediate water level height, and there is a need for simple models for generation of the most characteristic non-Gaussian characteristics, namely the asymmetric distributions of water level height and wave slope.

A simple class of non-linear oscillators can in fact reproduce some of the characteristic features of water wave processes and linear or non-linear response to ocean waves:

$$x''(t) - h(x(t), x'(t)) = \sigma B'(t), \tag{1}$$

where $B'(t)$ is Gaussian white noise. To construct an example with the typical asymmetry we start with the linear oscillator with mass m, stiffness k and damping c driven by a random force, $mx''(t) + cx'(t) + kx(t) = \sigma B'(t)$. To obtain asymmetry we let stiffness and damping depend on x, x', by letting the response frequency $\omega = \sqrt{k/m}$ depend on $x(t)$ and $x'(t)$, keeping the relative damping $\zeta = \frac{c}{2\sqrt{mk}}$ constant. Thus $\omega = \omega(t)$ varies between two limits, $\omega_1 < \omega_2$, and for simplicity we combine $x(t)$ and $x'(t)$ into a linear function to get $\omega = \omega(x, x') = \omega_1 + (\omega_2 - \omega_1)G(ax + bx')$, where G is a continuous function increasing from 0 to 1, while a and b are constant parameters. A positive value of a implies peaked waves with horizontal asymmetry, while the sign of b determines the vertical asymmetry. A negative b-value will make the distribution of the slopes negatively skewed.

The numerical ideas used for calculation of crest heights in Gaussian waves, can be used also in this non-Gaussian case. In the talk I shall describe the Slepian model and explain the use of the regression approximation for level crossing distances, wave periods and amplitudes. The generalization of the method to a non-linear oscillator is described.

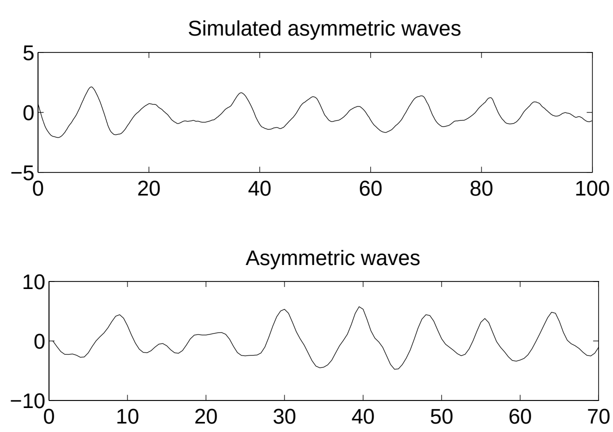

Figure 1: A simulation of the solution to (1) compared to wave data from the North Sea.