Effects of High-Order Nonlinear Wave-Wave Interactions on Deep-Water Waves

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1 Introduction

It is widely known that sea surface elevations of deep-water waves could be assumed as a stationary, ergodic and weakly nonlinear process. However, deep-water gravity waves suffer from and are sensitive to resonant interactions with the third order nonlinearities(*e.g.* Zakharov 1968) even if the nonlinearities are weak. Benjamin and Feir (1967) found that the Stokes waves are unstable for side band disturbances at the 3rd order nonlinearities. However, only a few investigations have so far been made effects of the higher order nonlinearities more than the 3rd order for random waves.

The aim of this study is to make clear the effects of the high order nonlinearities on the random waves from the results of numerical simulations solving.

2 Computational Method

The numerical simulations of random waves in 2D and 3D deep-water domains are carried out for linear, 2nd order and high order nonlinear solutions, more than 3rd order. The high order nonlinear simulations are performed by using a high-order spectral method developed by Dommermuth and Yue (1987). Time stepping of the surface profile and velocity potential are integrated by the fourth-order Runge-Kutta Gill method in the Fourier space. The initial surface profiles are comprised of the Wallops spectrum and phase constant of the Fourier components are given by the uniform random number. The initial statistics of the waves are given by $k_p a$=0.15 and m=5, 10, 30, 40, 60 and 100.Here, k_p is wavenumber of the spectral peak mode, a is a half of the significant wave height and m is a parameter controlling the spectral band width.

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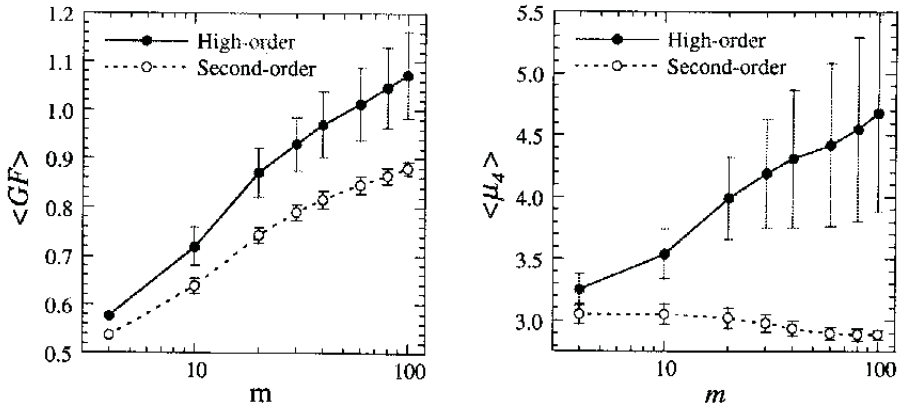
3 Results and Discussion

The results of the numerical simulations show that the temporal evolutions of the Fourier mode amplitudes of the high order nonlinear waves dramatically change during the propagation process, although the 2nd order nonlinear solution shows only weak oscillation of the spectra form due to the 2nd order harmonics.

The surface wave profiles suggested that the high order nonlinear interactions generate a single and outstanding wave like a freak wave, that is merely observed in the North Sea. The differences of skewness between the high order solution and 2nd order one are negligible. However, the high order nonlinearities exert remarkable influences upon the value of GF and kurtosisμ_4(Figure 1).



(a) GF vs m (b) mu_4 vs m

Figure 1 Influences of nonlinearities and spectral band width on wave statistics.

4 Conclusion

It could be concluded that i)the high order nonlinearities affect to large amount of energy exchange of the Fourier component during propagation process, ii) the influence of high order nonlinear interactions are negligible for the run length, but significant for the GF and kurtosis. Hence, the third order nonlinearities must be taken into considerations for the prediction of the maximum wave height and modeling freak wave.

References

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