# The Empirical Mode Decomposition and the Hilbert Spectra to Analyze Embedded Characteristic Oscillations of Extreme Waves

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## 1 Introduction

Fourier analysis has become the most valuable tool in spectral data analysis due to its ability and simplicity and has consequently been applied to all kinds of data of any scientific discipline, though, it is strictly limited to linear systems and stationary data series [e. g. Titchmarsh, 1948]. To avoid these constraints the Wavelet analysis has been developed [e. g. Daubechies, 1992]. Liu [2000 a, 2000 b] applies the continuous Morlet Wavelet Transformation to investigate the spectral energy content of ocean data series. Recently, Huang *et al.* [1998, 1999] developed the Hilbert Huang Transformation (HHT) to decompose any time-dependent data series into its individual characteristic oscillations with the so-called Empirical Mode Decomposition (EMD). Applying the Hilbert Transformation to any of these disintegrated Intrinsic Mode Functions (IMF) subsequently provides the Hilbert amplitude spectra with significant instantaneous frequencies. This technique generates distinct time-dependent spectra and, therefore, presumes entirely new physical insights of nonlinear and non-stationary processes of extreme waves. Schlurmann *et al.* [2000 a] investigate laboratory generated extreme waves and utilize an improved numerical algorithm of the HHT to decompose these data into their characteristic oscillations [Schlurmann *et al.*, 2000 b].

The present paper mainly concentrates on the application of the HHT to analyze extreme waves observed in the Sea of Japan.

## 2 Results and Discussions

Extreme waves are defined as transient waves existing in one specific location in one particular instant in time. Watersurface elevations were measured by three individual operating ultrasonic type wave gages off the coast of Yura, Japan [Mori *et al.*, 2000].

The central part of the Hilbert Huang Transformation is the EMD, a numerical sifting process to decompose a signal into its fundamental intrinsic oscillatory modes. The sum of all IMF match the signal very well and, therefore, completeness of this analyzing method is assured in principle. The orthogonality is satisfied in a practical sense, although it is not guaranteed theoretically. The definition of orthogonality seems to be global, the real meaning applies only locally that two components are orthogonal within a certain period of time.

Figure 1 presents the EMD of the watersurface elevations of one particular extreme wave event measured on 14-Dec-1988. The signal is disintegrated into ten IMF in total - the first seven IMF are shown here. This particular extreme wave event is separated into locally non-overlapping time scale components with varying amplitudes. It is clearly identified that each IMF is dominated by an almost constant inter-wave component with small intra-wave modulations. The first three IMF carry most of the embedded energy. Figure 2 presents results from the Wavelet and Hilbert Transformation. The top *panel a* contains the original recorded data series. Correspondingly, *panel b* shows the Wavelet (Morlet) spectrum and *panel c* presents the Hilbert spectrum.

Apparently, the Hilbert spectrum provides more distinct information on the time-frequency contents of this extreme wave event.

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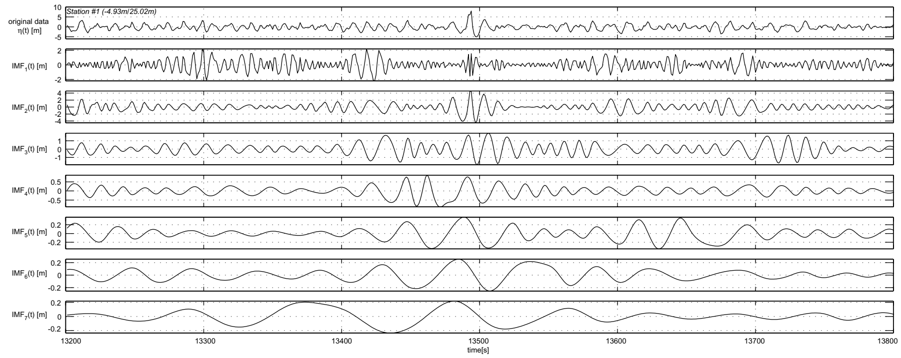


**Fig. 1.** EMD of watersurface elevation

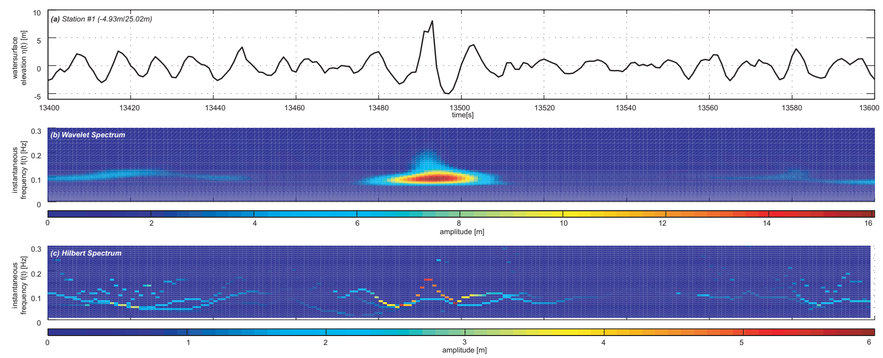


**Fig. 2.** Wavelet and Hilbert spectrum

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