NEWWAVES, SOLITONS AND SPREADING

Paul H. Taylor

Department of Engineering Science,
University of Oxford,
Parks Road, Oxford OX1 3PJ, U.K.
paul.taylor@eng.ox.ac.uk

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If rogue waves exist they must be inherently nonlinear. They apparently occur more often than would be predicted using the tail of the Rayleigh distribution for crest elevation - even after allowing for the obvious 2^{nd} order crest-trough asymmetry of steep waves. Such Stokes 2^{nd} order models are only weakly nonlinear – the kinematics at the free-surface are different for crests and troughs and there is the radiation-stress driven return flow beneath localised wave steep groups. However, the underlying dynamics are linear – the position of the dominant Fourier components for any time can be predicted using simple linear dispersion.

So long as only a 2^{nd} order accurate model is used, then the properties of extremes in an underlying effectively linear random process are simple to describe. The average shape of a tall crest tends to the scaled auto-correlation function (Lindgren, Boccotti, Tromans etc.), and this has sometimes been called NewWave. One of the reasons why this is an attractive model is that it connects the (averaged) properties of extremes to the power spectrum for an entire sea-state. This model is convenient for both physical experiments and computation as it gives a localised wave group consistent with a realistic spectrum such as JONSWAP.

Theoretical models of steep waves beyond 2^{nd} order in wave steepness have to account for nonlinear dispersion if they are to be accurate over long distance and time scales. The simplest example of this is the amplitude dependent Stokes correction to the dispersion of regular waves which arises at 3^{rd} order in the theory. At 3^{rd} order for narrow-banded wave groups, nonlinear evolution equations can be used to explore the effects of wave amplitude in the dynamics. The consequences are profound – and different for group structure in a longitudinal down-wave direction or structure laterally along the wave crests. The simplest NLEE is the nonlinear Schrodinger equation. For isolated uni-directional wave groups on deep water described by the positive (NLS+) version of the equation, there are soliton solutions where linear dispersion is balanced by 3^{rd} order amplitude dispersion to produce permanent (and robust) localised groups. Both one-dimensional physical experiments and computer modelling show that non-linearity cooperates with linear focussing when an extreme event is produced by having long waves overtake short waves. A localised wave group is formed which is more compact, taller and longer lasting than would be predicted by simple linear theory (even if 2^{nd} order corrected). Indeed, if the spectral components in the converging group are chosen suitably, a group forms which at least locally closely approximates a soliton. Perhaps such a soliton could be a good candidate for a model of rogue waves.

In contrast to uni-directional wave groups (structure along the mean wave direction but crests running transversely to infinity), variation in the height of a wave group along the wave crest direction leads to the NLS– or defocusing version of the equation. Non-linearity is still just as important but the linear focussing of components to produce an extreme event is weakened. Now there are no isolated soliton solutions. Thus we see that group structure in the mean wave and transverse directions has very different consequences for the production of extreme events.

Both versions (NLS+ and NLS-) of the Schrodinger equation in 1-D are solvable by inverse-scattering techniques. The combined spread sea (x,y,t) version is not. Neither is the directionally spread deep-water wave version of the NLS with the dynamical effect of the return current included – the Davey-Stewartson equation. Further, it is known that new physics enters with the extra spatial dimension – the resonant interactions of Phillips and Hasselmann. This suggests that basing the analysis of field data on one-dimensional models may be inappropriate. However, these evolution equations represent only an important and useful simplification of the full Euler equations for waves.

An efficient and robust numerical scheme for the solution for the full Euler equations for water waves has recently been derived (Bateman, Swan and Taylor). This is a new pseudo-spectral scheme based on the Dirichlet-Neumann (G-) operator of Craig and Sulem (1993). This operator permits the accurate conversion of the velocity potential on the free surface into the normal gradient of this velocity potential. This step is essential in any time-integration scheme for water waves based on the exact dynamic and kinematic boundary conditions. The scheme is restricted to non-overturning waves: the surface elevation has to be a single-valued function of the horizontal co-ordinates. Thus, only the early stages of wave breaking can be studied. The main advantages of the scheme are its efficiency and robustness. It can be run on a PC and is sufficiently robust that it can be used to predict the time histories and local kinematics of near breaking waves. The results are in excellent agreement with the high quality wave basin data recently reported by Johannessen and Swan. Examples of fully non-linear wave groups on deep water will be discussed in the talk, showing that the introduction of a realistic degree of directional spreading has dramatic consequences for the focussing of a steep wave group.

A fundamental problem of numerical modelling is the choice of a suitable initial condition to start the simulation. This is where the NewWave idea helps. Analysis of measured wave data in severe storms in the northern North Sea shows that the average shape of large waves in the open sea is well modelled by the scaled auto-correlation function (Jonathan and Taylor) after allowing for 2^{nd} order effects. Away from the focus point such a localised wave group would disperse. If the group is isolated, it disperses back to a linear background. Thus a suitable initial condition for numerical simulation would be a NewWave well before the focus time.

Using a realistic wave spectrum and directional spreading to define a NewWave well before (linear) focus, we follow the fully nonlinear evolution in time. The wave-wave interactions so important for uni-directional wave groups are significantly modified by directional spreading. In particular, the peak surface elevations arising in directional spread focussed wave groups are similar to those predicted by simple 2^{nd} order theory if the spreading is as large as that commonly assumed in severe winter storm waves. However, the underlying dynamics of spread group focussing are not close to linear – in the vicinity of the focus event, the group becomes more compact in the mean wave direction but wider in the along-crest direction. Thus, the shape of the wave group is modified – and for engineering applications the local factor on in-line wave kinematics to allow for spreading would be closer to unity than expected from linear theory. Furthermore, this localisation of the total energy into a small area of sea for a relatively short time has a permanent effect even as the group diverges to infinity - the directional spreading of the group far downstream is different to that far upstream.

All this work supports the contention that the evolution of directionally spread wave groups is fundamentally different to the evolution of one-dimensional wave groups. Thus, efforts to explain the occurrence of rogue waves should incorporate directional spreading.

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