Some experiences in estimating long and short term statistics for extreme waves in the North Sea

[Observations of "rogue waves" at North Alwyn during 1994-1998]

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This note sets out the results of probability analysis of "rogue" wave data recorded at North Alwyn between August,1994 and June, 1998. The results are presented as a bivariate distribution of normalised $H' = \frac{H}{H_{m0}}$ vs. $S' = \frac{S}{2\pi H_{m0} / gT_{zu}^2}$ for "rogue waves" defined as

$\{ H \mid H' > 2.0 , \& S' > 0.5 \}$.

A total of 114 rogue waves, based on this definition, were identified from a total of 394537 individual waves recorded during storms exceeding 6m significant height.

Table 1: Hs/Tz bins showing total number of waves (upper) and number of "rogue" waves (**lower**) in each bin. August 1994 - June1998

		Mid-points of Hs bins (Metres)						
Mid-point of Tz Cells ((Sec)		**6.5**	**7.5**	**8.5**	**9.5**	**10.5**	**11.5**	**12.5**
	7.5	48298 **21**	1865 **3**	1928 **8**	337 0	962 0	160 0	0 0
	8.5	154708 **38**	35729 **10**	4823 **6**	979 0	703 0	1001 0	429 0
	9.5	20154 **1**	41897 **15**	26715 **5**	4613 **4**	1607 0	1019 0	123 0
	10.5	953 0	5160 0	14226 **2**	10032 **1**	4362 0	2751 0	1371 0
	11.5	856 0	1394 0	1169 0	1939 0	1062 0	0 0	0 0
	12.5	390 0	391 0	391 0	0 0	0 0	0 0	0 0
	All Tz	225359 **60**	86436 **28**	49292 **21**	17900 **5**	8696 0	4931 0	1923 0

Table 1 was obtained by aggregating the individual up-cross wave data for each twenty-minute observation period into the corresponding Hs/Tz bin. The binned data were then analysed for the occurrence of rogue wave events in terms of the number of exceedences. The complete unbinned data were also analysed to determine the joint normalised height/steepness distribution of the rogue waves shown in Figure 1. The joint distribution was based on the simplified Weibull distribution of the normalised height and steepness for all waves in a bin given by Wolfram et al. (2000).

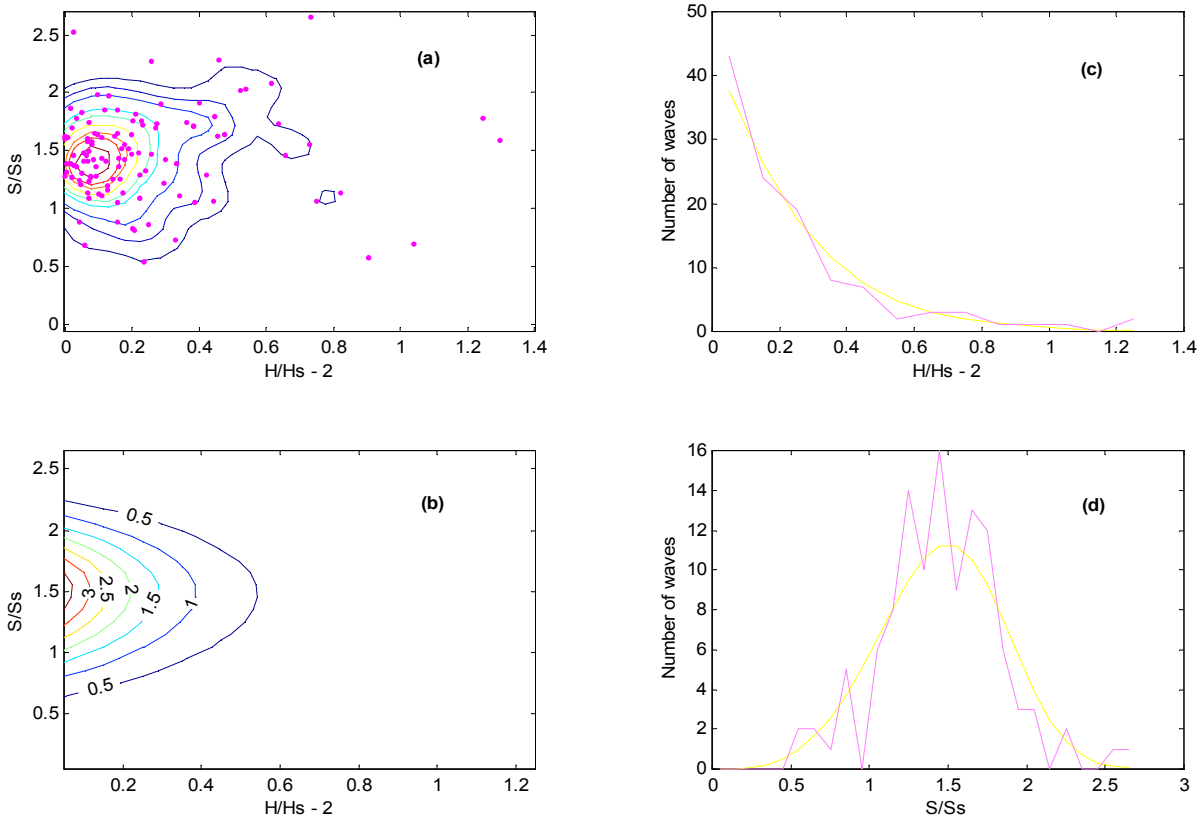
$$p(H', S') = \frac{\gamma H'^{\gamma-1}}{\eta^{\gamma}} \exp\left(-\left(\frac{H'}{\eta} \right)^{\gamma} \right) \frac{\beta S'^{\beta-1}}{\alpha^{\beta}} \exp\left(-\left(\frac{S'}{\alpha} \right)^{\beta} \right) \quad (1)$$

where α, β and γ are dimensionless parameters, and $\eta = \tau S^{\lambda}$ where τ and λ are also dimensionless parameters. The right tail of this distribution for $H' > 2.0$ can be simplified to

$$p(H', S' \mid H' \rangle\ 2.0 \,\&\, S' \rangle\ 0.5) = \frac{C}{\eta} \exp\left(-\frac{H' - 2}{\eta} \right) \frac{\beta S'^{\beta-1}}{\alpha^{\beta}} \exp\left(-\left(\frac{S'}{\alpha} \right)^{\beta} \right) , \quad (2)$$

since log likelihood fitting to the rogue data gives $\gamma \approx 1$ and where C is a normalising constant. The remaining parameters are $\alpha = 1.60$, $\beta = 4.18$, $\tau = 0.25$ and $\lambda = -0.089$.

Figure 1: Joint distribution of normalised waveheight *H'* and steepness *S'* for rogue waves. (a) scatterplot and kernel density estimate contours at 90% to 10%. (b) fitted distribution (c) marginal distribution of waveheight and (d) marginal distribution of steepness showing fitted model and data.



Kernel density estimates were made for the scatterplots for the height and steepness of the rogue waves against the waves immediately preceding and succeeding them. Quantile-quantile plots were also constructed to determine if the rogue waves departed from the expected distributions of wave height and steepness of their immediate neighbours. The KDE and QQ plots show that approximately 90% of the largest waves occur in groups where the immediately preceding or succeeding waves exceed 50% of Hs and 50% exceed Hs. The QQ plots show that the highest 20% of the rogue waves, i.e those with H'>2.3, follow a different normalised height distribution to their neighbours.

We conclude that the rogue waves were generally 50% steeper than the significant steepness and that the preceding and succeeding waves had steepness values around half the corresponding significant values while their heights were around the significant height. The rogue waves exhibited a marginal exponential distribution of normalised height, the marginal distribution of normalised steepness was Weibull and the joint distribution showed that the maxima of the conditional distribution of normalised steepness were constant at 1.5. Tentatively we propose that a more logical definition for "rogue" waves would be those where H'>2.3 since the probability distribution of these waves appears to be different from that of their neighbours.

Reference

Wolfram,J., Linfoot, B., and V. Venugopal. ,2000, Some results from the analysis of metocean data collected during storms in the northern North Sea. *Underwater Technology*. **24** (4).