CALCULATION OF NET SHAPES BY THE FINITE ELEMENT METHOD WITH TRIANGULAR ELEMENTS

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SUMMARY
The calculation of net shapes is performed using the finite element method. A triangular element is developed for the modelling of the net. The main hypothesis is that the strain in each triangular element is constant, the net has only two directions of twine and the twines are elastic. The forces due to the tension in twines are described and calculated by a direct method. Results of the model based on such triangular elements are given. The comparison with a model where each twine is described as an elastic bar is quite good. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS net; finite element method; direct formulation

INTRODUCTION
Nets are used in fishing especially in trawling. Net makers and fishermen want to know if the net is working well or if the stress in the net is acceptable during fishing.

Theret1 and Bessonneau2 developed a numerical method which describes each twine of the net by an inextensible bar. This method grouped a large number of twines in each numerical bar. The forces in this method are the drag due to the water flow, and also the weight and the buoyancy of the net. An iterative method gives the equilibrium shape of the net.

Ferro3 uses nearly the same method but he takes into account the elasticity of twines. He uses the finite element method, where elements are bars.

Tronstad and Larsen4 developed a membrane element used by the finite element method. The element has four corners and each side is parallel to the diagonal of net meshes. The deformation along each side of the membrane element can vary and the element stays plane. This element is two-dimensional, contrary to the previous methods, which use one-dimensional elements, i.e. bars.

The drawback of these methods is that the bars must be parallel to the net twines and the sides of the membrane element parallel to the diagonal of the net meshes, so the user is not free for the discretization of the net. To avoid this problem we have developed a triangular element. The definition of the triangular element is quite simple because the sides of triangles can be not parallel to the twines or to the diagonal of the net meshes. The refinement is simple because an element can be divided into several elements. To avoid the use of the stress–strain relation of the net, needed in the variational formulation, we use the direct formulation, i.e. forces are directly calculated from the position of the vertices of triangular elements.

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HYPOTHESIS AND METHOD

The net is split into triangles, each of them being able to cover a large number of meshes. Each vertex, or node, is linked to the net, so when the equilibrium position of each node is found, the equilibrium position of the net is found. The nodes are not necessarily located on net knots (cf. Figure 1, node 5).

In this method only diamond and square meshes can be modelled, because our method is able to introduce only two directions (called \( u \) and \( v \)) of twines in the net. In each triangle, all twines of one direction are parallel, i.e. all the \( u \) twines are parallel, and the same for the \( v \) twines. That leads to a constant longitudinal strain for the \( u \) twines, and the same for the \( v \) twines. The strain of the \( u \) twines can be different from that of the \( v \) twines. Obviously there is a variation of twine orientation and strain from a triangle to an adjacent one. All twines belonging to a triangle are modelled as bars made of isotropic and elastic material.

With these hypotheses, for each triangle, a relation between the forces applied to each node and their positions (\( X \)) can be found. For all the triangles, the force applied to each node (\( F \)) is the sum of the forces from each triangle. Thus a relation \( F(X) \) can be established. To calculate the equilibrium of a net, we must introduce the data into the model. In most of the cases of nets, the initial data give an initial position which is not in equilibrium, i.e. \( F(X_{\text{initial}}) \neq 0 \). To get the equilibrium position \( X_{\text{final}} \), where \( F(X_{\text{final}}) = 0 \), we use the Newton–Raphson iterative method:

\[
X_{k+1} = X_k - \frac{F(X_k)}{F'(X_k)}
\]

where \( X_{k+1} \) is the node position at iteration \( k + 1 \), \( F(X_k) \) is the force on nodes at iteration \( k \), and \( F'(X_k) \) is the derivative of the force relative to the position (or stiffness) on nodes at iteration \( k \).
Thus, at each iteration \( k \), the position is known \((X_k)\), so \( F(X_k) \) and \( F'(X_k) \) are calculated and the position \( X_{k+1} \) is deduced. A large displacement of the net can be obtained in this usual way (Desai et al., Zienkiewicz et al., Muttin).

FORCES APPLIED TO NODES

The net shape is due to all the forces applied to the net. We will focus our work on forces resulting from the tension and stiffness of twines, because these are the most significant. Some other forces are described by Theret.

In each triangle, the longitudinal strains of twines in the same direction are identical, i.e. all lengths of mesh sides along \( u \) twines are \( n \) and \( m \) along \( v \) twines. We assume that the longitudinal strain is small, so that linear theory applies. We also assume that the elasticity modulus \((E \text{ in Pa})\) and the section \((A \text{ in m}^2)\) of twines are constant.

In order to calculate more easily the forces on each node, we turn the triangle in the plane \( XOY \). In this plane, each node has Cartesians co-ordinates but also co-ordinates in number of meshes (cf. Figures 1 and 2).

Under these conditions, the distances between the nodes 1 & 2 and 1 & 3 can be expressed with the Cartesian co-ordinates and with co-ordinates in number of meshes:

\[
\begin{align*}
  x_2 - x_1 &= (u_2 - u_1)nx + (v_2 - v_1)mx \\
  y_2 - y_1 &= (u_2 - u_1)ny + (v_2 - v_1)my \\
  x_3 - x_1 &= (u_3 - u_1)nx + (v_3 - v_1)mx \\
  y_3 - y_1 &= (u_3 - u_1)ny + (v_3 - v_1)my
\end{align*}
\]

where \( x_1, \ldots, y_3 \) are Cartesian co-ordinates of the nodes (m), \( u_1, \ldots, v_3 \) are co-ordinates in number of meshes, and \( nx, ny (mx, my) \) are components along the \( X \) and \( Y \) axes of the \( u \) (v) mesh side (m).

The components of the mesh sides \((nx, ny, mx, my)\) are calculated with the four previous equations.

The lengths of the mesh sides are:

\[
\begin{align*}
  n &= \sqrt{nx^2 + ny^2} \\
  m &= \sqrt{mx^2 + my^2}
\end{align*}
\]

The tension in twines \((Tu, Tv \text{ in N})\) can now be deduced (cf. Figure 2):

\[
\begin{align*}
  Tu &= EA \frac{n - n_0}{n_0} \\
  Tv &= EA \frac{m - n_0}{n_0}
\end{align*}
\]

where \( n_0 \) is the unstretched length of the mesh side along the two directions \( u \) and \( v \) (m).
Now we calculate the force on each side of the triangle due to the twine tension. For example, the component along the $X$ axis of the force applied to the side of the triangle between the nodes 1 & 2 and due to the tension of the $u$ twines is

$$F_{xu}^{12} = (v_2 - v_1)Tu \frac{nX}{n}$$

where $v_2 - v_1$ is the number of twines $u$ acting on the side 1–2, and $nX/n$ is the part of the force acting along the $X$ axis.

The same components can be calculated along the three sides of the triangle, along the $X$, $Y$ axis and for the $u, v$ twines. That means 12 components or 'side forces':

$$F_{yu}^{12} = (v_2 - v_1)Tu \frac{nY}{n}$$

$$F_{xv}^{12} = -(u_2 - u_1)Tv \frac{mX}{m}$$
These 12 ‘side forces’ can now be distributed among nodes. Since the twines are uniformly spread along sides of the triangle (cf. Figure 2), we can divide each ‘side force’ between the nodes of this side. Thus $F_{12}^{12}$ is divided by 2 between nodes 1 and 2. The force on each node is the sum of all the components acting on this node. For example, the force component on node 1 along the $X$ axis is

$$f_x^1 = \frac{F_{31}^{31}}{2} + \frac{F_{31}^{12}}{2} + \frac{F_{31}^{31}}{2}$$

That gives

$$f_x^1 = (v_2 - v_3)Tu_{nx}^{nx} - \frac{(u_2 - u_3)Ty_{mx}^{mx}}{2m}$$

That means six components for the triangle in the plane XOY. The components along the $Z$ axis (perpendicular to the triangle) are zero, because these components are perpendicular to the twines

$$f_y^1 = (v_2 - v_3)Tu_{ny}^{ny} - \frac{(u_2 - u_3)Ty_{my}^{my}}{2m}$$

$$f_z^1 = 0$$

$$f_y^2 = (v_3 - v_1)Tu_{ny}^{ny} - \frac{(u_3 - u_1)Ty_{my}^{my}}{2m}$$

$$f_z^2 = 0$$

$$f_y^2 = (v_3 - v_1)Tu_{ny}^{ny} - \frac{(u_3 - u_1)Ty_{my}^{my}}{2m}$$

$$f_z^2 = 0$$

Thus, at this step, we have calculated the components of forces applied to the three nodes of the triangle, due to the tension of twines.

In order to calculate the forces on each node more easily, we had turned the triangle in the plane XOY. Now the triangle is turned back, to get the forces in the actual orientation of the triangle.

Up to now, only the forces applied to each node in one triangle have been calculated. To get the general forces applied to each node, the force components from all triangles are added. To complete the loading description, the other forces such as weight and drag (Theret$^1$) are added. These other forces are generally simple to calculate because they are independent of node positions.

VERIFICATION

The initial data introduced in the model are the initial positions of nodes and the description of each triangular element. This description gives for each triangular element the three nodes which
are the vertices, the mesh size, the twine stiffness, the number of meshes of each node and the other loadings (weight, drag, etc.).

Figure 1 shows a simple verification example with four triangular elements. The four boundaries of the net are fixed; only the internal node can move. The equilibrium position of this node must be exactly at the centre of the panel, because there is the same number of meshes between this node and each of the four corners of the net. The calculated equilibrium shows that the internal node is at the centre of the panel — more exactly, the distance to the centre is less than $10^{-9}$ m. The criterion of convergence is that the mean of residue of force on each node is less than $10^{-5}$ N. It took four iterations to achieve the convergence.

In this example, the twine stiffness is 10,000 N, the unstretched mesh side is 0.160 m, and the length of the boundary is 2 m. The calculated stretched mesh side is 0.189 m.

The accuracy of our method is also verified by comparing our results with those obtained from a reference model. The net panel implemented is square and is constituted by 1600 meshes and 3281 knots. The twine stiffness is 10,000 N; its diameter is 0.01 m and the mesh size is 1.2 m. The length of the top boundary is 32 m and the density of the net material is 2000 kg/m$^3$. The net is held by its top boundary and it is tightened by its own weight. The criterion of convergence is that the mean of residue of force on each node is less than 0.01 N.

Our model uses 1050 triangular elements and 512 nodes with one symmetry plane (Figure 3). The calculated shape is on the right side of Figures 3 and 4. Figure 3 shows the triangular elements, while Figure 4 shows the twines in each triangular element.
The comparison is made with a reference model which models each mesh side of the net using bars, as described by Ferro.\textsuperscript{3} The reference model constitutes 3136 twines and 1625 nodes with one symmetry plane (Figure 4, left hand side). The calculated shapes obtained by the two models are similar.

**RESULTS**

This method is applied to a bottom trawl, which consists of 12 panels of net and 18 cables. The net comprises more than 130,000 meshes and 260,000 knots. The shape is calculated for a trawler speed of 1 m/s and a depth of 100 m. The model constitutes 2232 nodes, 180 bars for modelling the cables and 4173 triangular elements for the net. The model has one vertical plane of symmetry. Figure 5 shows the calculated shape of the trawl.

![Figure 5. Top and side view of the calculated shape of a bottom trawl. Triangles represent the net and lines represent the cables](image)
CONCLUSION

The development of a triangular element for the calculation of nets is based on several hypotheses. The main ones are that the elongation and elasticity modulus are constant in each triangular element. The direct formulation gives the forces on each node. The equilibrium of the net is calculated by using the Newton–Raphson method.

The verification of the results by comparing those using the triangular element with results coming from the model using bars for each twine shows that the triangular element gives a reasonably good description for calculations of net shapes.

The main advantage is that it is easy to make a local refinement without refining all the net. In fact a triangular element can be divided into three triangles by adding a node. Therefore triangular elements can be small in an area of high gradient of deformation, and can be large in an area of low gradient of deformation. Therefore the number of nodes can be much smaller than in other models and time calculation also much smaller.

REFERENCES