FEM modeling of flexible structures made of cables, bars and nets

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ABSTRACT: A Finite Element Method (FEM) modeling devoted to flexible structures made of nets, cables and bars, such as fishing gears and fish farming cages is described. The paper focuses on a specific triangular element devoted to net modeling. The main hypothesis is that the net twines remain straight in each triangular element. The twines tension is described in mathematical form. The virtual work principle is used to calculate forces on the 3 vertices of the triangular element from the twines tension. The other forces taken into account in this model are weight in water, mesh opening stiffness, twines contact, bending stiffness, catch pressure, and hydrodynamic loading. Equilibrium is found using the Newton-Raphson method. Results are given for trawls made of diamond and hexagonal meshes, for cod-end with grid and catch and for floating fish cage.

1 INTRODUCTION

Users of flexible structures such as trawls or fish farming cages, generally want to know the shape of such structures to assess, for example, the loss of volume of the fish cage in the current or the possibility for small fishes to escape from the trawl. Such assessment can be carried out by tests in tank, but numerical models are more and more used.

Flexible structures such as trawls or fish farming cages are made of netting, cables, chains, bars, floats, dead weights. Such components can be regarded as surface element (netting), linear element (cables, bars and chains) or point element (floats, doors and dead weights).

This paper focuses mainly on net modeling because linear and point elements have been largely studied (Zienkiewicz 1989, Desai 1972).

Few net models devoted to nets exist:

- Theret (1993), Bessonneau (1997), Niedzwiedz (2001), Pashen (2004), Lee (2004), Tsukrov (2003) have developed 3D numerical methods which describe twines of the net by numerical bars. These methods take into account a large number of twines in each numerical bar. The forces considered are the drag due to the water flow, but also the weight and the buoyancy of the net. Some of these methods take into account the twine elasticity. An iterative method gives the equilibrium shape of the net. Ferro (1988) uses nearly the same method but he uses the finite element method, where elements are bars. The drawback of these models is that the numerical bars must be parallel to the actual twines of the netting, that means that the user of the modeling is not free for creating the numerical bars.

- Tronstad and Larsen (1997) have developed a membrane element used by the finite element method in 3D. The element has 4 corners and each side is parallel to the diagonal of net meshes. The deformation along each side of the membrane element can vary and the element stays plane. This element is a two-dimensional element, contrary to the previous methods which use one-dimensional elements: i.e. bars. The drawback is near the same as the previous: the sides of membrane element must be parallel to the diagonals of the actual meshes, once again the user is not free for creating the membrane elements.

- O’Neill (1997) has developed a model for axi-symmetrical structures, such as trawl cod-end. It is a 2D model. He takes into account the twine tension, as previously, but also the mesh opening stiffness and the pressure of the fish catch on the net. The drawback of this modeling is that it is devoted to only axi-symmetrical structures.

To avoid the problem of constrained numerical elements and of axi-symmetry hypothesis, and to take into account further mechanical behaviors, a Finite Element Method (FEM) model of the net based on a triangular element has been developed (Priour 1997, 1999, 2001, 2003). The triangle is the simplest shape to describe a surface element, it is why it has been chosen.

The FEM model takes into account the inner twines tension, the drag force on the net due to the current, the pressure created by the fish in the cod-end, the floatability and weight of the net, the mesh opening stiffness and the bending stiffness of the net.
The FEM model is able to describe all the net and cables, which means that: – for a trawl, the cod-end, the wings, the headline and also the rigging up to the boat are taken into account, and – for a fish farming cage, the floats, the netting, the dead weights and the mooring lines are taken into account. The net is modeled by triangular elements and cables, warps, bridles and bars are modeled by linear element (bars).

2 HYPOTHESIS AND METHOD

Due to the high number of knots in structures made of netting, the time required for the determination of the equilibrium position of knots is generally too long. Thus, in the FEM model described in this paper, the net is split into triangular elements, each of them being able to cover a large number of meshes (Figure 1). They can be also smaller than a mesh (Figure 7): there is no constraint. The triangles are contiguous, which means that a vertex can belong to several triangles, and triangle boundaries are not necessary parallel to the twines or to the meshes diagonal. So, the user is free to create the triangular elements. The refinement is easy: a triangular element can be divided in 3 triangles without any modification on the remaining. Each vertex is linked to the net, so when the equilibrium position of each vertex is found, the equilibrium position of the net is found.

In this FEM model the diamond and square meshes can be modeled, as well as hexagonal meshes. Such nets have 2 directions of twines (called here u and v) for diamond and square netting or 3 directions for hexagonal one.

For diamond and square mesh netting (2 directions of twines), each direction of twine is kept parallel in each triangular element: in other words, all the u twines are kept parallel, just as the v twines (Figure 2). The hypothesis is the same for 3 directions of twines. This leads to a constant deformation for each direction of twines, that means that all the u twines have the same deformation, just as the v twines. Obviously there is a variation of orientation and deformation from one triangular element to an adjacent one. This hypothesis is reasonable if all the elements are small.

With these hypotheses the force on each vertex can be calculated. These forces depend on the position of vertices. From these forces the Newton-Raphson iterative method gives the equilibrium position of the net.

3 TWINES TENSION

Here the forces due to twines tension are described for diamond and square mesh netting, the method is similar for hexagonal meshes.

All the twines of a triangular element are modeled as a bar made of isotropic and elastic material. The material can have 2 elastic modulus, one in traction and one in compression (very low or zero), to take into account the un-stretched twine.

In each triangular element the elongation of twines of a same direction is constant. Thus, mesh sides along u twine have the same length (| u |) and all mesh sides along v twine have the same length (| v |). The tension is $T_u$ (N) along the u direction and $T_v$ along the v direction:

$$ T_u = E A \frac{|u| - n_0}{n_0} $$

$$ T_v = E A \frac{|v| - n_0}{n_0} $$

Where $E$ is the twine modulus of elasticity (Pa), $A$ is the twine section (m²), $n_0$ is the un-stretched length of mesh side (m) and $|u|$ ($|v|$) is the stretched length of mesh side along u (v) direction (m).
In order to calculate the tensions, the stretched length of mesh sides must be calculated. These two lengths are calculated with the following equations (Figure 2):

\[ S_{12} = (V_2 - V_1) \mathbf{v} + (U_2 - U_1) \mathbf{u} \]  
\[ S_{13} = (U_3 - U_1) \mathbf{u} + (V_3 - V_1) \mathbf{v} \]

Where \( U_1, U_2, U_3, V_1, V_2 \) and \( V_3 \) are the mesh number of the three nodes (as marked on Figure 2), \( S_{12} \) (\( S_{13} \)) is the vector from node 1 to node 2 (1 to 3) and \( \mathbf{u} \) & \( \mathbf{v} \) are the mesh side vector of \( u \) and \( v \) twines.

With these two previous equations the mesh side vectors can be deduced:

\[ \mathbf{u} = \frac{V_2 - V_1}{n} \quad S_{12} - \frac{V_3 - V_1}{n} \quad S_{12} \]  
\[ \mathbf{v} = \frac{U_3 - U_1}{n} \quad S_{12} - \frac{U_2 - U_1}{n} \quad S_{13} \]

Where \( n \) is the number of mesh sides in the triangular element:

\[ n = (U_3 - U_1)(V_2 - V_1) - (U_2 - U_1)(V_3 - V_1) \]

The forces on each vertex are calculated with the principle of virtual work. For example, \( f_{x1} \) the X component of the force on vertex 1 is assessed by a displacement \( (d_{x1}) \) along X axis of the vertex 1. This displacement leads to an external work for the triangular element:

\[ W_e = f_{x1} \quad d_{x1} \]

This displacement \( (d_{x1}) \) induces a variation of twines length \( (d|\mathbf{u}| \& d|\mathbf{v}|) \) and consequently an internal work for the triangular element:

\[ W_i = (T_u \quad d|\mathbf{u}| + T_v \quad d|\mathbf{v}|) \quad n \quad \frac{n}{2} \]

Due to the principle of virtual work the internal work must be equal to the external one. Which gives:

\[ f_{x1} = \left( T_u \quad d|\mathbf{u}| + T_v \quad d|\mathbf{v}| \right) \quad n \quad \frac{n}{2} \]

That gives the forces due to twines tension on the 3 vertices:

\[ f_i = (V_3 - V_2)T_u \quad \frac{\mathbf{u}}{2|\mathbf{u}|} + (U_3 - U_1)T_v \quad \frac{\mathbf{v}}{2|\mathbf{v}|} \]

\[ f_z = (V_2 - V_1)T_u \quad \frac{\mathbf{u}}{2|\mathbf{u}|} + (U_2 - U_1)T_v \quad \frac{\mathbf{v}}{2|\mathbf{v}|} \]

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4 SUM OF THE FORCES

Up to now, the forces applied to each triangular element are calculated. These forces have been spread on each node (vertices of each triangle and extremities of bars for cables and chains). The force components of all triangles and bars are added to get the whole forces applied to each node. The complete loading is the sum of forces which depend on tension, drag, buoyancy, weight, twines contact and catch:

\[ \mathbf{F} = \mathbf{F}_e + \mathbf{F}_d + \mathbf{F}_t + \mathbf{F}_w + \mathbf{F}_c + \mathbf{F}_o + \mathbf{F}_b \]

Where \( \mathbf{F} \) is the total loading on net and bars, \( \mathbf{F}_e \) is due to the twine and cable tension (described previously), \( \mathbf{F}_d \) is due to the drag force on twines and bars, \( \mathbf{F}_t \) is due to the buoyancy of net and bars, \( \mathbf{F}_o \) is due to the weight of net and bars, \( \mathbf{F}_d \) is due to twines contact, \( \mathbf{F}_c \) is due to the catch, \( \mathbf{F}_o \) is due to the mesh opening stiffness, and \( \mathbf{F}_b \) is due to the bending stiffness (\( \mathbf{F}_e, \mathbf{F}_o \) and \( \mathbf{F}_b \) have been implemented in the FEM model only for diamond and square mesh nets).

5 METHOD OF NUMERICAL CALCULATION

Forces (\( \mathbf{F} \)) are calculated on each node. These forces depend on the node positions (\( \mathbf{X} \)). Thus the relation \( \mathbf{F}(\mathbf{X}) \) is established. Obviously, in most of the net shape calculations, the initial position \( \mathbf{X}_{\text{initial}} \) of the nodes is not at equilibrium so \( \mathbf{F}(\mathbf{X}_{\text{initial}}) \neq \mathbf{0} \). To achieve the equilibrium position \( \mathbf{X}_{\text{final}} \), where \( \mathbf{F}(\mathbf{X}_{\text{final}}) = \mathbf{0} \), the Newton-Raphson iterative method is used:

\[ \mathbf{X}_{k+1} = \mathbf{X}_k - \frac{\mathbf{F}(\mathbf{X}_k)}{\mathbf{F}'(\mathbf{X}_k)} \]

Where \( \mathbf{X}_{k+1} \) is the nodes position vector at iteration \( k + 1 \) (m), \( \mathbf{F}(\mathbf{X}_k) \) is the force vector on nodes at iteration \( k \) (N), and \( \mathbf{F}'(\mathbf{X}_k) \) is the matrix of derivative of force (or stiffness) on vertices at iteration \( k \) (N/m).

A large displacement of the structure (net and cable) can be obtained by this usual method (Desai and all 1972, Zienkiewicz and all 1989, Muttin 1991). It consists in applying total force \( \mathbf{F}(\mathbf{X}) \) and stiffness \( \mathbf{F}'(\mathbf{X}) \) to revise the co-ordinates \( \mathbf{X} \) of nodal points. At every stage total force and stiffness depend on the new position of nodes. The process is repeated until the total force is very small.
The numerical method used here is currently used in finite element method.

6 VALIDATION

6.1 Hydrostatic pressure

Comparisons are carried out between cod-end shape measurement under well-known condition and the corresponding FEM model. O’Neill et al. (1997) described a test, which consists of a cod-end hanging under gravity. The catch force \( F_c \) (in equ. 14) is due to the pressure of the catch on the net. The pressure implemented in the FEM model is:

\[
P = \rho g h
\]

Where \( P \) is the pressure of the catch on the net (Pa), \( \rho \) is the mass density of the catch (Kg/m\(^3\)), \( g \) is the gravity (9.81 m/s), and \( h \) is the height of the catch (m).

The parameters of the tested case are:
- Mesh size = 37.2 mm,
- Number of meshes around = 50,
- Number of meshes long = 50,
- Volume of catch = 0.0265 m\(^3\),
- Mass density of the catch = 1000 Kg/m\(^3\),
- Radius of the top ring = 25 cm.

In the FEM model the cod-end is made of 742 nodes, 1360 triangular elements, 1 rope to close the net and 2 planes of symmetry.

Figure 3 shows that the FEM model gives a satisfactory description of the cod-end. This description is about the formulation of forces due to the twine tension and to the hydrostatic pressure (\( F_c \) and \( F_e \) in equ. 14).

6.2 Net tightened by its weight

The FEM model described in this paper is verified by comparing the results with those obtained from a reference model.

The net panel calculated by the FEM model and by the reference model is square and is constituted by 40 \( \times \) 40 meshes and 3281 knots. The twine stiffness is 10 000 N, its diameter is 0.01 m and the mesh size is 1.2 m. The length of the top boundary is 32 m and the density of the net material is 2 000 Kg/m\(^3\). The net is held by its top boundary and it is tightened by its own weight.

The FEM model uses 1050 triangular elements and 512 nodes with one symmetry plane. The criterion of convergence is that the mean of residual of force on each node is less than 0.01 N. The calculated shape is on the middle and bottom of the Figure 4.

The reference model uses numerical bar for each mesh side, as described by Ferro (1988). The reference model uses 3136 bars and 1625 nodes with one symmetry plane (Figure 4, top). The criterion of convergence is the same as previously.

The calculated shapes obtained by the FEM and the reference models are similar.

6.3 Pressure of the caught fish

Results carried out by the FEM model are compared with flume tank tests. The flume tank tests were conducted on cod-end partly filled with bags of water (Premecs-I 2000). The implemented pressure used in the catch force (\( F_c \) in equ. 14) is:

\[
P = \frac{1}{2} \rho C_d V^2
\]

Where \( P \) is the pressure of catch on the netting (Pa), \( \rho \) is the mass density of water (kg/m\(^3\)), \( C_d \) is the drag coefficient, and \( V \) is the flow velocity (m/s).

For this purpose the distance between the front of the catch and the extremity of the cod-end was introduced in the FEM model for each test. This distance has been measured during flume tank tests.

Figure 5 shows the results of the model (net) and of the flume tank tests (cross). This comparison shows that the numerical model provides a good description of the cod-end.
7 APPLICATION

7.1 Trawl

The Figure 6 represents the calculated shape of a trawl from the doors to the cod-end. The trawl is made of netting panels of diamond meshes and hexagonal meshes (Van Marlen 1980).

The trawl consists of 4 panels of hexagonal mesh net in the front part, 36 panels of diamond mesh net and 36 cables. More than 1 000 000 meshes and 2 000 000
knots are used in the netting. The shape is calculated for a trawler speed of 2.08 m/s.

The structure is constituted by 1129 nodes and 91 bars for modeling the cables, 1954 triangular elements for the diamond mesh, and 81 triangular elements for the hexagonal mesh. The trawl holds 10 m³ of fish catch. The structure has one vertical plane of symmetry.

7.2 Bending stiffness

The FEM model is used to assess the bending stiffness (EI) of a netting.

The panel tested has only 9 meshes. The mesh side is 0.069 m long. The twine diameter is 4 mm. The mesh opening stiffness is 0.009 N.m/Rad. The neutral angle

Figure 7. On the top, photo of a netting bent on its own weight. Calculation of the bent panel: the twines are drawn on the middle and the triangular elements on the bottom.

Figure 8. Calculation of a cod-end with a grid: complete view on the bottom, few netting panels have been hidden on the top and on the middle. The bold lines are the frame of the grid.

Figure 9. Design plan of the circular cage. The bottom netting panel is on the middle surrounded by side panels.
between twines is 0.62 Rad (36°). These two parameters are used by \( F_a \) in equ. 14. The panel is hold partly horizontally, the other part is bent under its own weight (Figure 7).

The panel in the FEM model is made of 273 nodes and 491 triangular elements. The criterion of convergence is when the mean force on nodes is less than \( 1.1 \times 10^{-6} \) N. The EI coefficient is adjusted to find the same deformation as during the test. That gives \( 450.10^{-6} \) Nm\(^2\). This value is close to the one assessed by Sala (2004) who found values between \( 470.10^{-6} \) and \( 690.10^{-6} \) Nm\(^2\).

7.3 **Grid in a cod-end**

The cod-end is made of 92 meshes around and 97 meshes along. The mesh side is 72.5 mm. The grid is 1.75 m by 1.23 m. The rectangular frame and the driving netting panel behind the grid is visible on the top of the Figure 8. This panel drives out the small fish. Its mesh side is 30 mm. The towing speed is 1.5 m/s and the catch is 2 m\(^3\).

The cod-end in the FEM model is made of 1235 nodes, 77 bars for the grid and cables, and 2442 triangular elements for the netting.

The centre view of Figure 8 shows the gap between the grid and the netting which allow the larger fish to pass through.

7.4 **Circular cage**

The cage is made of 12 netting panels around and one panel for the bottom (Figure 9). The diameter of the cage is 20 m, there is 1200 meshes around and 300 meshes high. The mesh side is 0.035 m long and 1.2 mm thick. 1200 Kg of dead weights are spread at the bottom of the cage. The mooring is made of 3 chains of 150 m long of 10 Kg/m. The 3 buoys are 3 m high and 2.3 m\(^3\) of volume. The current is 0.5 m/s.

The cage in the FEM model is made of 165 bars and 1400 triangular elements. The calculation of the circular cage is shown on Figure 10.

8 **CONCLUSION**

For the calculation of the shape of flexible structures made of netting, cables and bars, a numerical model based on the finite element method has been developed. This FEM model is a 3 dimensional one, based on a surface triangular element that represents the behavior of elastic net. These triangular elements have no constraint relatively to the netting: the sides are not necessary parallel to the actual twines or to the diagonal of meshes. That leads the user free to model the structure. The FEM method gives pretty good comparisons with tests.

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