“LONG-TERM STATISTICS WITH EQUIVALENT STORM MODELS, FOR EXTREME VALUES OF SIGNIFICANT WAVE HEIGHT”

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Introduction

The following presentation is divided in two parts:

1) storm analysis carried out by applying Equivalent triangular Storm Model and by using as input data significant wave height time series given by buoys both of RON(Rete Ondametrica Nazionale) network and of NOAA-NDBC(National Data Buoys Center). Sensibility of this model to different sampling $\Delta t$ between two consecutive records (varies between 1 and 6 hours) has been investigated. The effects of the results for long term statistics have been investigated.

2) directional analysis of storms carried out by using as input data significant wave height and wave direction time series given both by buoys and by wave model. The aim of this analysis has been the evaluation of variability of direction during storms to introduce a directional storm definition.
Definition of sea storm

A sea storm is a sequence of sea states in which the significant wave height $H_s$ is above a certain, constant, threshold $H_s'$ and does not fall above it for a certain time interval $\Delta t$ (Boccotti, 2000). The values of the threshold $H_s'$ and of the time interval $\Delta t$ depend on the characteristics of the recorded sea states and, thus, on the location under study. Boccotti (2000) has proposed the following values:

$$H_s' = 1,5 \bar{H}_s$$

$$\Delta t = 12 \text{ hours}$$
**Wave data**

To execute the analysis of sea storms were used, as input data, *significant wave heights* $H_s$, given by buoys of NOAA-NDBC network (National Oceanic and Atmospheric Administration’s – National Data Buoy Center, USA) are considered. The historical wave data consists in significant wave height, peak and mean period, given with a sampling between two records equal to one hour. In the paper the NOAA buoys 46006, is considered in the Pacific Ocean and the 42001, in the Atlantic Ocean (Gulf of Mexico).
Wave data
Equivalent Triangular Storm model

In both cases the analysis was carried out by applying the Equivalent Triangular Storm model, which associates a triangle to each actual storm by means of two parameters:

- the triangle height \( a \) that gives the storm intensity and is equal to the maximum significant wave height during the actual storm;

- the triangle base \( b \) that represents the storm duration and it is such that the maximum expected significant wave height is the same both in the equivalent triangular storm and in the actual storm.

\[
H_{\text{max}}(a,b') = \int_0^\infty \left( 1 - \exp \left[ \frac{b'}{a} \ln \left( 1 - P(H; H_s = h) \right) \right] \right) dh dH
\]
Equivalent Triangular Storm

Severest actual storm and the relative ETS storm at 42001 buoy with different $\Delta t$ varying among 1 and 6 hours.

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Equivalent Triangular Storm

Severest actual storm and the relative ETS storm at 42001 buoy with different $\Delta t$ varying among 1 and 6 hours.

- $a = 11.1\ m$, $b = 50\ hours$
- $a = 10.31\ m$, $b = 100\ hours$
- $a = 10.31\ m$, $b = 130\ hours$

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Equivalent Triangular Storm

Severest actual storm and the relative ETS storm at 46006 buoy with different $\Delta t$ varying among 1 and 6 hours.

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Equivalent Triangular Storm

From figures it is evident that, considering a time interval between two consecutive records greater than 1 hour, the peaks of the storms is not always well identified. The maximum significant wave height can be smaller than that one actually occurred at the apex of the storm. This difference increases as greater $\Delta t$ is. The duration grows as $\Delta t$ increases.

Severest actual storm and the relative ETS storm at 46006 buoy with different $\Delta t$ varying among 1 and 6 hours.

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**Base-height regression**

Considering all the sea storm have been evaluated the base-height regressions of triangular storms, for both locations considered and for time interval between two consecutive records equal to 1, 3, 6 hours:

\[
a_{10} = \frac{\sum_{i=1}^{N} a_i}{N}
\]

\[
b_{10} = \frac{\sum_{i=1}^{N} b_i}{N}
\]

\[N = 10 \ n_{years}\]

\[n_{years} = \text{number of years of recording}\]

Example of base-eight regression in a dimensionless form.

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Base-height regression

We observe that, both for Atlantic and Pacific Ocean, important differences on the storm duration are identified as $\Delta t$ varies, observing the minimum duration of the sea storm at the smallest $\Delta t$. For a fixed sea storm, its duration grows as $\Delta t$ increases.
Probability of exceedance of significant wave height for all wave direction

Then, for both locations considered, we have obtained the probability of exceedance of significant wave height for all wave direction $P(H_s > h)$, which represents the probability that the significant wave height is greater than a fixed threshold $h$ at a certain location. Such probability is well represented from our wave data with a three-parameter Weibull distribution. Defining the distinctive parameters $u$, $h_l$ and $w$ of the Weibull distribution, the exceedance probability may be written as:

\[
P(H_s > h) = \exp \left( - \left( \frac{h - h_l}{w} \right)^u \right)
\]

which is defined for $H_s > h_l$.

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The return values and mean persistence over the threshold

As consequence of the previous result it is considered of interest to evaluate, in the same cases of the analysis of the storms, the return period $R(H_s > h)$ of a storm in which the maximum significant wave height is greater than $h$, and the mean persistence $D_m(h)$ above the threshold. It is defined as:

$$R(H_s > h) = \frac{\tau}{N(h, \tau)}$$

Large time interval

Number of triangles during $\tau$ with height greater than $h$

$$N(h, \tau) = N(\tau) \int_h^\infty p_A(a) da$$

Number of triangles during $\tau$

Probability density function of triangles height

$$p_A(a) = -\frac{\tau}{N(\tau)} a \frac{dp(H_s = a)}{da}$$

$p(H_s = a) = -dP(H_s > a)$

Base-height regression for ETS

$$R(H_s > h) = \frac{\bar{b}(h)}{h p(H_s = h) + P(H_s > h)}$$
Mean persistence over the threshold

The mean persistence $D_m(h)$ represent the mean time interval during which $H_s$ is above the threshold $h$ in the storms in which this threshold is exceeded.

\[ D_m(h) = \frac{\tau P(H_s > h)}{\tau/R(H_s > h)} \]

- time interval during $\tau$ in which $H_s > h$
- number of storms during $\tau$ with $H_s > h$

\[ D_m(h) = P(Hs > h)R(Hs > h) \]
The return values and mean persistence over the threshold

Return period \( R(H_s > h) \) and mean persistence \( D_m(h) \) for different time intervals between two consecutive records (46006 buoy).

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The return values and mean persistence over the threshold

Return period $R(H_s > h)$ and mean persistence $D_m(h)$ for different time intervals between two consecutive records (42001 buoy).

In both cases we can observe that for fixed value of the threshold of significant wave height the corresponding return period and mean persistence grow as time interval $\Delta t$ between two consecutive records increases.
The return values and mean persistence over the threshold

<table>
<thead>
<tr>
<th>46006 R(anni)</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
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<tbody>
<tr>
<td>h(R)[m] deltat=1h</td>
<td>12.5</td>
<td>13.2</td>
<td>13.9</td>
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<td>15.5</td>
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<tr>
<td>D_{m}[hours]</td>
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<td>2.5</td>
<td>2.4</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>h(R)[m] deltat=3h</td>
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<td>12.6</td>
<td>13.4</td>
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<td>15</td>
</tr>
<tr>
<td>D_{m}[hours]</td>
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<tr>
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<td>12.2</td>
<td>12.9</td>
<td>13.9</td>
<td>14.6</td>
</tr>
<tr>
<td>D_{m}[hours]</td>
<td>9.5</td>
<td>8.8</td>
<td>8.2</td>
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<table>
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<td>h(R)[m] deltat=1h</td>
<td>7.8</td>
<td>8.5</td>
<td>9.3</td>
<td>10.4</td>
<td>11.2</td>
</tr>
<tr>
<td>D_{m}[hours]</td>
<td>6.8</td>
<td>6.4</td>
<td>6</td>
<td>5.6</td>
<td>5.4</td>
</tr>
<tr>
<td>h(R)[m] deltat=3h</td>
<td>7.6</td>
<td>8.4</td>
<td>9.2</td>
<td>10.4</td>
<td>11.3</td>
</tr>
<tr>
<td>D_{m}[hours]</td>
<td>12.6</td>
<td>11.8</td>
<td>11.1</td>
<td>10.3</td>
<td>9.8</td>
</tr>
<tr>
<td>h(R)[m] deltat=6h</td>
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<td>7.7</td>
<td>8.5</td>
<td>9.6</td>
<td>10.5</td>
</tr>
<tr>
<td>D_{m}[hours]</td>
<td>22.8</td>
<td>21.3</td>
<td>20</td>
<td>18.5</td>
<td>17.5</td>
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Return value of significant wave height $h(R)$ and mean persistence $D_{m}(h)$ for fixed values of return period $R$, at both considered location.

From the table we can see that for a fixed value of return period $R$ the return value of significant wave height $h(R)$ decreases as $\Delta t$ increases, while the corresponding mean persistence over the threshold $D_{m}(h)$ increases.
Directional analysis (in collaboration with Professor Carlos Guedes Soares, CENTEC, Instituto Superior Tecnico, Lisbon)

A directional analysis of storm has been carried out by processing data given both by directional buoys and by wave model. Some locations have been considered in Atlantic and Pacific Ocean, in North and Mediterranean Sea. Variability of direction during severe storms has been investigated by using as input data significant wave height and wave direction time series. All the storm in the data set have been identified with Boccotti criteria and analyzed by using Equivalent Triangular Storm Model. For each storm identified variability of direction during storm has been investigated. Starting from the results of the above analysis a directional definition of storm has been introduced. Finally base-height regression for fixed sectors have been evaluated to calculate directional return period of a storm in which the maximum significant wave height is greater than a fixed threshold $h$ and wave direction within a well defined sector.
**Directional analysis**

The results of three location chosen as representative of wave direction trends during storms will be shown: Figueira da Foz which is characterized by only one direction from which the strongest storm occur, Mazara del Vallo characterized by two directions and one point in North Atlantic Ocean by three directions.
Directional analysis

Mazara del Vallo
Directional analysis

Figueira Da Foz
Atlantic Ocean

Mazara Del Vallo
Mediterranean Sea

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Directional analysis

Figueira Da Foz
Atlantic Ocean

Mazara Del Vallo
Mediterranean Sea

(5)
North
Atlantic
Ocean

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Directional definition of storm

Figueira da Foz
Atlantic Ocean

North Atlantic Ocean

Mazara del Vallo
Mediterranean Sea

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Directional analysis

Figueira da Foz

North Atlantic Ocean

Mazara del Vallo

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Directional analysis

Figueira da Foz

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**Directional analysis**

Scatter plots showing the relationship between average direction during storms and standard deviation of direction for different threshold values of wave height. The plots are labeled with 'threshold=0.5Hs max', 'threshold=0.7Hs max', 'threshold=0.9Hs max', and 'threshold=Hs max'.

**Mazara del Vallo**

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Directional analysis

North Atlantic Ocean

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Directional analysis

\[
P(H_s > h; \theta_1 < \theta < \theta_2) = \exp\left[-\left(\frac{h - h_i}{W_{\alpha}}\right)^a\right] - \exp\left[-\left(\frac{h - h_i}{W_{\beta}}\right)^a\right]
\]

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**Directional definition of storm**

Starting from storm set identified with Boccotti criteria, we may define a storm as directional storm if

\[
0.5 H_{\text{smax}} < H_s < H_{\text{smax}}
\]

\[
\text{dir}_{1,2,3} - \Delta \vartheta < \text{dir} < \text{dir}_{1,2,3} + \Delta \vartheta
\]
Directional definition of storm

Mazara del Vallo
Mediterranean Sea

For storms with $H_{s,\text{max}} > 2h_{\text{crit}}$
Directional definition of storm

Figueira da Foz
North Atlantic Ocean

For storms with $H_{s \text{ max}} > 2h_{\text{crit}}$
**Directional definition of storm**

Starting from storm set identified with Boccotti criteria, we may define a storm as directional storm if

\[
\text{for } 0.5 H_{\text{smax}} < H_s < H_{\text{smax}}
\]

\[
\text{dir}_{1,2,3} - \Delta \vartheta < \text{dir}_m < \text{dir}_{1,2,3} + \Delta \vartheta
\]
Figueira da Foz

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Mazara del Vallo

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For storms with $H_{s\text{ max}} > 2h_{\text{crit}}$

### Figueira da Foz

<table>
<thead>
<tr>
<th>$\Delta\theta$</th>
<th>N dir1</th>
<th>% $N_B(h_{s\text{ max}} \geq 2h_{\text{crit}})$</th>
<th>N dir1m</th>
<th>% $N_B(h_{s\text{ max}} \geq 2h_{\text{crit}})$</th>
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</thead>
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<tr>
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<td>7</td>
<td>5.1</td>
<td></td>
<td></td>
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<tr>
<td>$20^\circ$</td>
<td>40</td>
<td>29.2</td>
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<td></td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>76</td>
<td>55.5</td>
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### Mazara del Vallo

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<th>N dir1m</th>
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<td>77</td>
<td>37.7</td>
<td>107</td>
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### North Atlantic Ocean

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<th>N dir1m</th>
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<td>$30^\circ$</td>
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<th>N dir2m</th>
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Thank you very much for your attention