Evidences of the Existence of Freak Waves

Sverre Haver

Statoil, E&P Norway, N-4035 Stavanger, Norway
svha@statoil.com

Abstract. As an extended introduction, the consequences of a possible existence of freak waves is discussed from a risk point of view, where focus is on the risk for loss of human lives. Herein freak waves are defined as wave events which are not captured by a second order model for the surface process, which as of today is the most advanced wave model for routine engineering. Finally, a major part of the paper is devoted to a review of literature which have presented wave events which may be examples of freak wave events.

1 Introduction

Over the last 2-3 decades major improvements are made regarding the modelling of environmental conditions for the purpose of designing offshore structures. This is the case both when it comes to the understanding of the underlying environmental processes and even more when it comes to the availability of good quality data which has made an empirical modelling of the environmental conditions rather accurate for a number of locations world-wide. Of course the models are not perfect and there are obviously rooms for improvements e.g.:

- Modelling of current in deep waters.
- Simultaneous modelling of wind sea and swell. This is mainly a question of more data of such events.
- Improved joint probabilistic modelling of wind waves water level and current.
- Closed form crest height model valid for various levels of sea state steepness and water depths.
- Improved predictions of the most extreme weather conditions (in terms of mean characteristics), i.e. so called 10^{-4} - weather.
- Kinematics (particle velocity and acceleration) associated with real ocean waves.

Although challenging from an environmental point of view, these short comings do not represent major problems from a design point of view. Doing some sensitivity studies, conservative choices can be made and together with the load factors used in the design process, a safe design should be achieved. More challenging from a design point of view is the wave - structure interactions - this topic is associated with very large uncertainties and typically requires costly model tests in order to be solved properly.

One environmental problem, however, stand out as a possible major problem - freak waves - if it is proven that they exist as a separate population. There are a number of indications - more or less subjectively - indicating the existence of wave events which are much larger (either in terms of the wave height or in terms of the crest height) or much steeper than expected by the reporter. Of course - being realisations of a random process - there is always a possibility (although very small) that an unexpectedly large event is occurring. Assuming a 10-year storm is affecting an ocean area being so large that it can be divided into 100 sub areas between which extreme storm waves can be assumed to be statistically independent, one may well see a wave close to a 1000-year wave in one of these sub areas. If this is the reason for the observed unexpected large events, they do not represent a particular problem. The likelihood of occurrence are then baked into our standard design process.

For a structure to be designed for a site on the Norwegian continental shelf, the Norwegian authorities require that the following extreme load cases are controlled:

- Environmental loads corresponding to a return period of 100 years in combination with a load factor typically taken to be 1.3. No major damage are permitted for this load event. For a number of ocean structures the 100-year load is often reasonably well approximated by the loads caused by the 100-year wave.
- Environmental loads corresponding to a return period of 10000 years in combination with a load factor typically set to 1.0. Again - for a number of structures this load is reasonably well approximated with the load caused by the 10000-year wave. For this limit state local damage is accepted, but the situation shall not develop into a catastrophic event, i.e. the structure shall not collapse or sink. The latter is implemented by requiring that the structure in damaged condition (i.e. after being exposed to the 10000-year wave induced response) can withstand loads with a 100-year return period with the load factor typically set to 1.0.

When we are predicting 100-year and 10000-year loads, we account (at least within the Norwegian practise) for a certain deviation from a Gaussian surface process. And if observed unexpected large wave events can be concluded to be rare realisations of a slightly non-Gaussian surface process, an acceptable structural safety is tacitly assumed to be achieved by the limit states mentioned above.

It is, however, this authors point of view that we can not exclude the possibility that some of these observed unexpected large wave events are realisations from a separate freak wave population. The physical conditions that could onset such a population are not yet known and, accordingly, neither the relative frequency of occurrence of events. These events are most probably so rare that it is not likely to effect our predictions of 100-year wave induced loads. However, if existing, it may well impact our prediction of accidental wave induced loads, i.e. loads with a retum period in the order of 10000 years. If this is the case, this means that the load we presently adopts as a load with an annual probability of occurrence of 10^{-4} actually should be associated with an annual probability of occurrence of e.g. 10^{-3}. The load that should have been used as an accidental load, i.e. a load with an annual probability of occurring of 10^{-4}, could well be the load we (by excluding freak waves as a separate phenomenon) associate with a annual probability of 10^{-5}.

Let us illustrate the possible consequences of this by a simple but realistic example. At the Norwegian Continental Shelf, a jacket structure is usually designed such that the height from the still water surface to the deck level is so large that it is ensured that the wave crest height with an annual probability of occurrence of 10^{-4} does not reach the deck level. This means that the topside is not exposed to loading from the accidental wave, and, accordingly, the upper bay of the substructure is not designed to withstand major wave loading on the deck structure. If freak waves do exist for that particular site, one can well imagine that the actual 10000-year crest height is 10-20% larger than the accidental crest height according to which the necessary deck height is determined. This larger wave crest could submerge the lowest part of the deck structure with a couple of meters and this will result in an incredible horizontal load pulse. This load increase is not covered by our safety factors and a worst consequence is that the structure collapses in the upper bay. For a manned jacket this is a catastrophic scenario and the annual probability of such an event has to be extremely small.

A common measure for expressing the risk to people on board these platforms are the Fatal Accident Rate (FAR), which is defined as the expected number of fatalities per 10^8 exposed hours. For a given platform the FAR-value should in principle include all risks that represent a threat to the crew, i.e. explosions and fires, working accidents, collisions with other vessels, and structural failure due to weather and earthquakes. Let us for the sake of illustrations assume that an acceptable risk would be obtained by requiring $FAR < 5$. A good rule for a robust new structure would be to further require that structural failures due the environmental loading (wind, waves or earthquake) should represent a very small contribution to the FAR-value, say $FAR(environment) < 0.5$. There is a rather simple relation between the FAR-value and the annual probability of failure. Assuming all onboard lost due to the accidental environmental load, it is not too difficult to show:

$$\begin{aligned} p_f &= 10^{-5} \;\Rightarrow\; FAR \approx 0.1 \\ p_f &= 10^{-4} \;\Rightarrow\; FAR \approx 1 \quad . \\ p_f &= 10^{-3} \;\Rightarrow\; FAR \approx 10 \end{aligned} \tag{1}$$

Shall we fulfil the requirement above, it is seen from Eq. (1) that the resulting annual probability of structural failure has to fulfil:

$$p_f\,(environment) < 5\,10^{-5}. \tag{2}$$

This is to be implemented as an estimate of the actual failure probability. In carrying out a structural reliability analysis, effects of gross errors (human errors in a broad sense) are usually not modelled explicitly. Referring to the failure probability estimated through a straight forward structural reliability analysis as a nominal failure probability, one can well imagine that gross errors could cause the actual (or true) failure probability to be $3-10$ times larger: This would of course depend on how sensitive the failure mode under consideration is to gross errors and/or the efforts done through procedures and training to minimise the impact of such failures. Taking 5 as a reasonable error factor accounting for gross errors, this means that a reasonable requirement to the nominal annual failure probability could be:

$$p_{f,nominal}\left(environment\right) \lesssim 10^{-5} . \qquad (3)$$

As of today we will possibly have to interpret freak waves as some sort of a gross error. Ensuring that our structure fulfils Eq. (3) when excluding the freak wave phenomenon, one can hope that Eq. (2) is not too much violated if, in principle, freak waves could be consistently treated in a reliability assessment.

Eqs. (2 and 3) represent extremely rare events, but it is events at this probability that is of interest regarding structural failures. In order to verify that we actually can reach a target safety as low as indicated, we need to understand phenomena corresponding to such low annual probabilities of occurrence. If a freak wave population do exist, they will most probably affect our load predictions of such low probability events and, consequently, the risk exposure to crew and platform.

2 Definition of a Freak Wave

At present there is no broad consensus regarding what should be defined as a freak wave event. Over the years a ratio of wave height to significant wave height larger than 2 is taken as a definition of a freak wave. To this author this criterium is somewhat vague since nothing is said about the duration of the observation window, i.e. is it a 20min. time series, a 3-hour series, or is the observation window covering the whole storm event. Although the extremes are not extremely dependent of the time period, T, covered (roughly proportional with $\sqrt{ln\left(\nu_0^+\right)T}$, where ν_0^+ is the expected zero-up crossing frequency of the wave process), the expected ratio will vary somewhat whether one look at 20-min. events or the full storm length.

To this author it seems reasonable to define freak waves as something that is beyond the knowledge available for routine design. This means that the criterium will evolve with time as our understanding is improved. If and when the freak phenomenon is fully understood, there is no reason to continue referring to these waves as freak waves. They will then be the extreme waves a structure is supposed to be designed against at a certain annual probability level.

As of today, the best we can do regarding the surface process is to describe it as a second order process, i.e.:

$$X_2(t) = X_1(t) + DX_2(t) \quad \mathbf{(4)}$$

where, see e.g. [8]:

$$X_1(t) = \sum_{k=1}^{N} A_k \cos(w_k t + q_k) = Re\left[\sum_{k=1}^{N} B_k \exp(i w_k t)\right]. \quad \mathbf{(5)}$$

$Re[\cdot]$ denotes the real part of a complex number, and $B_k \exp(i w_k t)$ are the complex Fourier amplitudes. Furthermore, A_k and q_k are Rayleigh distributed and uniformly distributed, respectively. The mean square of A_k is related to the underlying wave spectrum, $s_{XX}(w)$, through:

$$E\left[A_k^2\right] = 2 s_{XX}(w) dw_k, \quad dw_k = w_k - w_{k-1}. \quad \mathbf{(6)}$$

The first order approximation to the surface process is given by Eq. (5), while the second order correction, $DX_2(t)$, can be written, see e.g. [13]:

$$DX_2(t) = Re\left[\sum_{m=1}^{N}\sum_{n=1}^{N} B_m B_n \left\{H_{mn}^{+} e^{i(w_m + w_n)t} + H_{mn}^{-} e^{i(w_m - w_n)t}\right\}\right]. \quad \mathbf{(7)}$$

The functions H_{mn}^{+} and H_{mn}^{-} are usually referred to as quadratic transfer functions and should be evaluated for all frequency pairs (w_m, w_n). Closed form solutions for the quadratic transfer functions are available see e.g. [13].

A part of a second order simulation is shown in Fig. 1. The underlying first order process and the second order correction are also shown. It is seen that the main effect of the second order correction is to make the troughs slightly shallower, the crests slightly higher and the wave front slightly steeper.

Such a second order model is available in a number of computer codes for load calculations. The surface shown in Fig. 1 is obtained using Wavemaker, [18]. The surface represented by such a model seems to be broadly accepted as a rather accurate model for real waves. The empirical distribution functions of 20-min. maximum crest heights and 20-min. minimum trough depths of the storm data included in [6] are compared with the corresponding distribution functions obtained under a first and a second order assumption, respectively, in Figs. 2 and 3. It is seen that the second order simulation results in a reasonable fit, except for the largest observed crest height. This particular observation will be discussed later on.

Regarding the calculation of the corresponding kinematics, things are more complicated and there is no general agreement on how this should be done in order to be fully consistent with the simulated surface process.

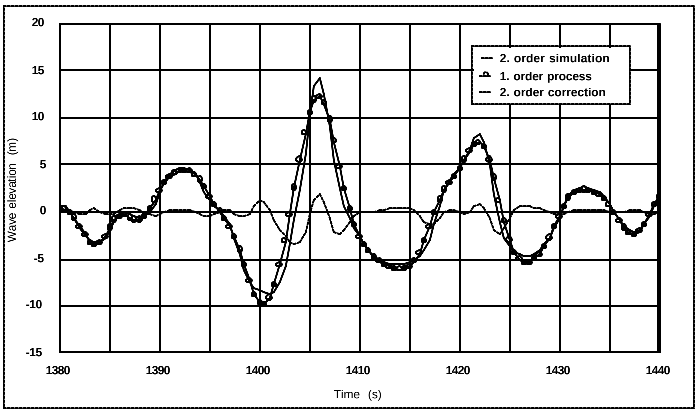

Fig. 1. Example of a simulated second order surface elevation process

Regarding crest heights, wave heights, and various measures of sea state steepness, a second order model is from this authors point of view today state-of-art. This is the background for suggesting the following definition of a freak wave event:

> *A freak wave event is an event (crest height, wave height, steepness, or group of waves) that represent an outlier when seen in view of the population of events generated by the second order model.*

If one is to look at freak waves in available data, one has to define the length of the observation window and a reasonably high fractile for the largest event in the actual window. This is discussed in [6]. There the length of the window is taken to be 20 min. since most available data series at least in Norway correspond to such a duration. In a second order process, the ratio of wave height to significant wave height that is likely to be exceeded in 1 out of 100 cases is about 2.0 while the same fractile for the crest height to significant wave height ratio is 1.25. Based on this, the following criterium is suggested as an indicator of possible freak wave events:

> *If for a 20-min time series,* $c_{max} / h_{m0} > 125$ *and/or* $h_{max} / h_{m0} > 2.0$, *the event is a possible freak wave. Further investigation will be necessary in order to finally conclude.*

Similar criteria can be established with respect to wave steepness and wave groupiness.

3 Some Observations of Possible Freak Waves

Over the years, a number of possible freak wave episodes are referred to in the literature. As a consequence of a number of episodes with significant wave induced damage on ships off the eastern South African coast during the fifties, sixties and early seventies, the subject of freak or abnormal waves received some attention during the seventies, see e.g. [11], [12], [1] and [3]. The definition of a freak wave adopted by WMO (The World Meteorological Organisation) reads, [1]:

> *"A freak wave may be defined as a wave of a considerable height ahead of which there is a deep trough. Thus it is the unusual steepness of the wave which is its outstanding feature and makes it dangerous to shipping. Reports so far suggest that such waves have usually occurred where a strong current flows in the opposite direction to a heavy sea."*

A couple of actual observations supporting this deep trough is referred to in [11]. The first is due to the Master of the Edinburgh Castle describing an episode taking place off the eastern South African coast. The vessel was heading into a strong south-west wind and a heavy south-west swell, but being 750 feet long and of 28000 gross tonnage, these conditions presented no serious problems to her. According to the master: "Under these conditions she was very comfortable for three-quarters of an hour or so. The distance from one wave top to the next was about 150 feet and the ship was pitching and scending about 10-15 degrees to the horizontal. And then it happened. Suddenly, having scended normally, the wavelength appear to be double the normal, about 300 feet, so that when she pitched she charged, as it where, into a hole in the ocean at an angle of 30^{o} or more, shovelling the next wave to a height of 15 or 20 feet before she could recover 'out of step'." (The wave lengths referred to above are surprisingly short. We will therefore question the unit feet - meter seems more likely.)

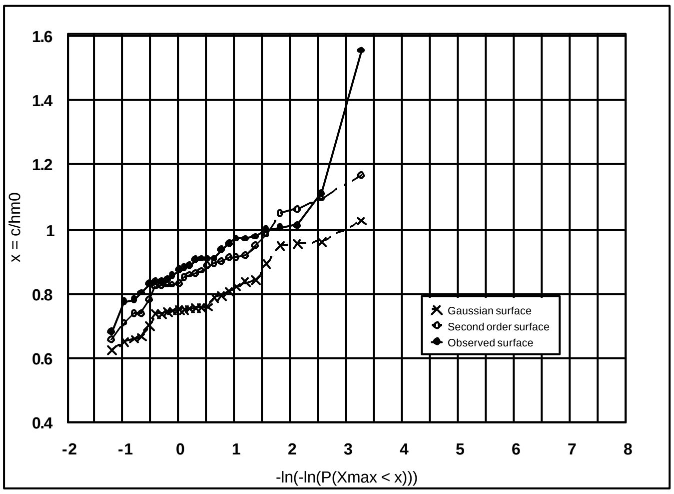

Fig. 2. Adequacy of the Gaussian - and second order assumption in representing observed 20-min. largest crest heights. (Normalised with the significant wave height, h_{m0}.)

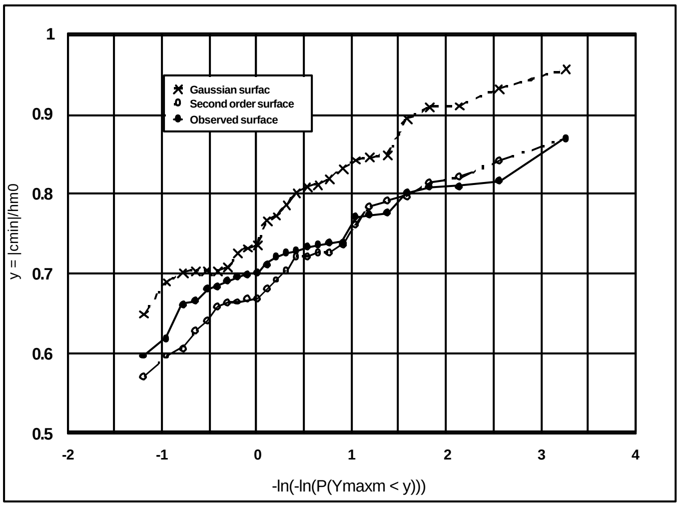

Fig. 3. Adequacy of the Gaussian - and second order assumption in representing observed 20-min. smallest trough depths. (Normalised with the significant wave height, h_{m0}.)

When this story reached the national press, it brought forward the following story from a second world war commander: "When I was serving at the cruiser Birmingham during the Second World War we had a similar experience in those waters one night which I recall the more vividly for being on watch at the time. We were about 100 miles south-southwest of Durban on our way to Cape Town, steaming fast but quite comfortably into moderate sea and swell when suddenly we hit the 'hole' and went down like a plummet into the next sea which came green over A and B turrets and broke over our open Bridge. I was knocked violently off my feet, only to recover and find myself wading around in 2 feet of water at a height 60 feet above normal sea level."

The idea of a long trough followed by a steep crest is elaborated in the paper by [12]. With a strong current opposing the waves, and the wave field built up by three sinusoidals of different lengths, such a scenario is illustrated within the framework of linear wave theory. For a sea state consisting of three pure swell systems, this model could possibly be representative. However, in most real sea systems, energy will be spread over a much broader range of frequencies and the probability for a very unfavourable phasing between the various components will be very small.

The early autumn of 1995, Queen Elisabeth II headed into a major storm, "Luis", off New Foundland, [2]. The maximum wave height from the ship's log was close to 30m. In a radio interview with the ship master after the storm, he refers to a particular episode where they from the bridge were looking at a wall of water for a "couple of minutes" before it hit the ship with some damage well above the water line. In the end of January 1995, a semi submersible, Veslefrikk B, operated by Statoil, was hit by a wave resulting in significant damage to the winch housing at the southern corner and the double bottom close to the western column. One of the crew members gave an oral description of this wave event that reminds very much of that of the QE II master. He also referred "to wall of water they could see for a couple of minutes". In both these episodes, the wave conditions were rather severe, and the particular wave events will not necessarily be of a freak type. The reason for including these observations is the phrasing "a wall of water for a couple minutes". Whether the couple of minutes should be two minutes or, say, 30s, is not a major point, but these observations, although not very scientifically documented by this paper, suggest that these very extreme waves appear for the observers more or less as a frozen profile for some time. Such a situation could not be maintained for several wave periods within a second order frame work.

Some few measurements of possible freak waves have been made over the years. Concentrating on the Danish sector of the North Sea, [15] present a number of cases involving waves we according to the criteria above will consider as possible freak waves. They show several cases where the crest height is about twice the significant wave height, i.e. a factor 2 as compared to the factor 1.2 introduced in this paper as a definition of a possible freak crest height. Even if we consider these as the largest out of 2000-3000 waves, we would consider these realisations to be well outside of what we would expect within a second order framework. Increasing the number of underlying waves from 100 to 2500, would suggest that the body of the extreme value distribution is shifted some 30 - 50% towards higher values. As the crest heights referred to above are measured at a water depth of about 40m, it is most likely that the wave profiles under extreme conditions are significantly affected by the bottom. [15], however, do also refer to some few episodes from deeper water. Of particular interest is a case from the Ekofisk field, where significant damage was reported more than 20m above still water level. The significant wave height at the time of the damage is not known, but a value in the order of 10-12m is reasonable, suggesting a crest height close to twice the significant wave height. At this depth, the bottom is expected to influence the wave profile of the largest waves, but it is not likely to be a governing effect.

Kjeldsen, [9], has considered data recorded from the Frigg field in the North Sea. The water depth is about 100m, and limited depth is not expected to be an important parameter regarding the surface elevation process. Some few observations seem to belong to a possible freak wave class, in particular when attention is given to the crest height. Yasuda *et al*., [20], have considered the occurrence of freak waves off Japan. They define a freak wave as a wave with a wave height being twice the significant wave height. A number of waves fulfilling this definition are included. An interesting example of a possible freak wave off Japan is shown by [19], but the measurements made off Japan correspond to rather shallow water, 43m, i.e. comparable to the depth of the cases from the Danish sector of the North Sea reported by [15]. We expect that results obtained for such water depths will not necessarily be representative for deeper waters, at least not if we consider sea states of a similar characteristic period.

A number of examples of heavy weather damages caused by giant waves are presented by [10]. In particular, he reefers to the capsizing of the semi-submersible, Ocean Ranger. The initiating event of this tragedy was according to [10] a giant wave that struck on the windows and flooded the control room. Over the years there has also been a number of ships disappearing without trace in stormy conditions with a considerable loss of human lives, an example being the Derbyshire disaster, see e.g. [5] for some further references to this event. The 4-year old bulker being 965 feet long disappeared in a severe typhoon some 500 miles south of Japan. Since vessels are lost without trace, they must have been sinking very fast. In reasonable weather conditions this seems to suggest that the initiating event is a major explosion. In very severe weather conditions, the impact with a giant wave seems just as likely. The structural strength may in some cases have been significantly degraded due to age and, possibly, lack of proper maintenance. Some ship designs may also be somewhat more vulnerable to severe consequences of possible heavy weather damage. In spite of all this, one can still not exclude that the reason for the loss of some of these ships is that they unfortunately hit a wave well beyond the expected design waves.

Faulkner and Buckley, [5], do also refer a number of other episodes where massive damage to ships due to giant waves is reported. In 1943, the liner "Queen Elizabeth" hit a trough preceding a giant wave off the east coast of Greenland. The wave broke over the ship and was followed by a second wave. The wave impacts shattered the bridge windows about 90 feet above the normal water line. The fore deck was smashed 0.15m below its normal level. In a contribution to a HSE-study, [7], Prof. Faulkner, in addition to the episode referred to above, also mention an episode in 1942 where "Queen Mary", carrying 15000 US troops onboard, was close to capsizing in steep elevated seas in the north east Atlantic.

January 1 1995, a Statoil operated jacket platform, "Draupner", was hit by a giant wave, see Fig. 4. The meteorological conditions in connection with this event is discussed by [17]. The water depth in the area is about 70m. The wave was measured by a down-looking laser device and the significant wave height averaged over a 20-

min. period was about 12m. The maximum wave height, see Fig. 5, was close to 26m, i.e. identified as a possible freak wave by the criterium established on basis of 20-min. measurements. The impressive thing about this wave, however, is its crest height which is measured to about 18.5m, well into the class of possible freak waves. This event is included in the data underlying Fig. 2 and it clearly deviates from the typical pattern. Within a second order frame work, Fig. 2 suggests that the probability of obtaining this value of the crest height to significant wave height ratio is 1:1000. The observed adjacent trough depth is less than 40% of the crest height. The conditional probability of having such a shallow trough following a major crest height is under the second order assumption most probably in the order of 1:100. Accordingly, the probability of observing an event like the event shown in Fig. 5 during a 20-min. window is in the order of 1:100000. For a typical North Sea site, the probability of observing a 20-min. sea state with a significant wave height of 10m or more is about $4 \cdot 10^{-4}$. Assuming 1 out of 10 to correspond to a steepness being necessary for producing the most extreme crest heights, we will expect about 1 event per year in at a given site. Due to the correlation between adjacent 20-min. events, a more proper interpretation may well be 10 events in a row every 10 years. No matter of interpretation, and in spite of the rather approximate probabilities presented in this paragraph, the observation of an wave event shown in Fig. 5 is rather unlikely event under the second order assumption. This author will take this event as an indication that the most extreme wave events are effected by phenomena not covered by our second order model.

The phenomena needed for explaining such a wave is not yet resolved. It could be an inherent energy fluctuation with a period much larger than 20 minutes (and a corresponding spatial distance) and thus destroyed by our assumption of stationarity. If that is the case, it will have major impacts on our fitting of probabilistic models since the ergodicity assumption will fail if the stationarity assumption fails. Another possibility is energy focusing as the wave system is travelling in space, see e.g. [16], [4], [14] or other disturbances suddenly make 3. and 4. order correction processes very important for a short time (maybe some few wave periods) and limited spatial area (of the order of some few wave lengths).

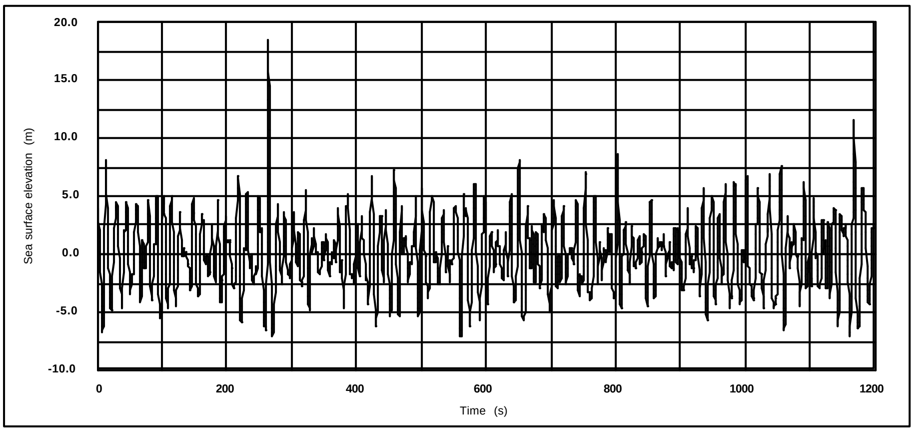

Fig. 4. A 20-min. wave recording at the "Draupner" platform, January 1, 1995 at 15:20

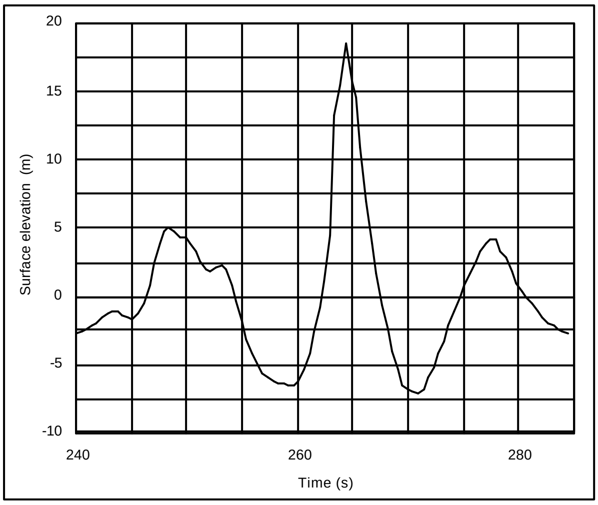

Fig. 5. The "New Year Wave" at "Draupner".

References

1. Atkins : Special Reports of Freak Waves. London Meteorological Office, London (1975). (The details on this reference is not known by the authors.)
2. Bigio : LUIS and the Buoys....and the Queen. 4th International Workshop on Wave Hindcasting and Forecasting", Banff (1995)
3. Dawson : Freak Ocean Waves Are Episodic. New Scientist (January 1977)
4. Dysthe, K.B., Trulsen, K.: Note on Breather Type Solutions of the NLS as Models for freak-waves. Physica Scripta, vol. T82 (1999)
5. Faulkner, D., Buckley, W.H.: Critical Survival Conditions for Ship Design. Proceedings for the International Conference; Design and Operation for Abnormal Conditions. Glasgow (1997)
6. Haver, S., Andersen, O.J.: Freak Waves - Rare Realisations of a Typical Population or Typical Realisations of a Rare Population?. Proceedings ISOPE-2000. Seattle (May 2000).
7. HSE: Review of Greenwater & Waveslam, Design & Specifications, Requirements for FPSO/FPU's, Offshore Technology Report - OTC 2000 004, Health & Safety Executive. Prepared by PAFA Consulting Engineering Ltd., London (March 2000)
8. Jha, A.K.: Nonlinear Stochastic Models for Loads and Responses of Offshore Structures and Vessels. PhD. Dissertation. Stanford University (1997)
9. Kjeldsen, S.P.: Breaking Waves. In: Tørum, A. and Gudmestad, O.T. (eds.): Water Wave Kinematics. Kluwer Academic Publishers, Dordrecht The Netherlands (1990)
10. Kjeldsen, S.P.: Examples of Heavy Weather Damage Caused by Giant Waves. Technomarine, no. 820 (1996)
11. London Meteorological Office:The One from Nowhere", Marine Observers, 35. London (1965)
12. Mallory : Abnormal Waves off the South African Coast - A Danger to Shipping. The Naval Architect (July 1975)
13. Marthinsen, T., Winterstein, S.R.: On the Skewness of Random Surface Waves. Proceedings ISOPE'92, San Francisco (1992)
14. Osborne, A.R.: The Random and Deterministic Dynamics of "Rogue Waves" in Unidirectional, Deep-Water Wave Trains. The Third International Workshop on Very Large Floating Structures (VLFS'99), Honolulu (1999)
15. Sand, S.E., Ottesen-Hansen, N.E., Klinting, P., Gudmestad, O.T. and Sterndorff, M.J.: Freak Wave Kinematics. In: Tørum, A. and Gudmestad, O.T. (eds.): Water Wave Kinematics. Kluwer Academic Publishers, Dordrecht The Netherlands (1990)
16. Stansberg, C.T.: Nonlinear Extreme Wave Evolution in Random Wave Groups. ISOPE'2000, Seattle (2000)
17. Sunde, A.: Kjempebølger i Nordsjøen (*in Norwegian*), Vær & Klima, vol. 18, no. 1 (1995)
18. Sweetman, B., Winterstein, S.R.: Second Order Random Ocean Waves: Prediction of Temporal and Spatial Variation from Fixed and Moving References - The Routine WAVEMAKER, Version 3.2. Rep. No. RMS-37. Civil Engineering Dep., Stanford University, Stanford (1999)
19. Tomita, H, Sawada, H.: Statistics of Heavy Weather Actual Ocean Wave Data in North Sea and Japan Sea with Reference to Abnormal Waves. ISOPE'99, Brest France (1999)

20. Yasuda, T., Mori, N., Nakayama, S.: Characteristics of Giant Freak Waves Observed in the Sea of Japan. Proceedings Waves'97 Ocean Wave Measurements and Analysis, Vol. 2. Virginia Beach (1997)