Wave Crest Sensor Intercomparison Study: An Overview of WACSIS

The Wave Crest Sensor Intercomparison Study (WACSIS) was designed as a thorough investigation of the statistical distribution of crest heights. Measurements were made in the southern North Sea during the winter of 1997–1998 from the Meetpost Noordwijk in 18 m water depth. The platform was outfitted with several popular wave sensors, including Saab and Marex radars, an EMI laser, a Baylor wave staff and a Vlissingen step gauge. Buoys were moored nearby to obtain directional spectra. Two video cameras viewed the ocean under the wave sensors and their signals were recorded digitally. The data analysis focused on comparisons of the crest height measurements from the various sensors and comparisons of the crest height distributions derived from the sensors and from theories. Some of the sensors had greater than expected energy at high frequencies. Once the measurements were filtered at 0.64 Hz, the Baylor, EMI and Vlissingen crest height distributions matched quite closely, while those from the other sensors were a few percent higher. The Baylor and EMI crest distributions agreed very well with the statistics from second order simulations, while previous parameterizations of the crest height distribution were generally too high. We conclude that crest height distributions derived from second order simulations can be used with confidence in engineering calculations. The data were also used in investigations of crest and trough shapes and the joint height/period distribution. [DOI: 10.1115/1.1641388]

Introduction

Knowledge of the statistical distribution of crest heights given the wave spectrum is central to the problem of setting deck heights. Unfortunately, there is still considerable uncertainty about the form of this distribution. The empirical evidence is confusing, since different types of instruments have tended to give different results. The theoretical problem is difficult since it is essential to account for the non-linearity of the waves.

The participants at the E&P Forum (1995) Workshop on Uncertainties in the Design Process felt that there was a need to compare wave sensors to determine which data sets were most reliable and develop an industry consensus on the form of the wave crest distribution. In response to this need, the WACSIS (Wave Crest Sensor Intercomparison Study) JIP was begun in 1997. The key to the experiment was to place all of the popular sensors on the same platform located where they were likely to experience large waves in one season. The location of the measurements was in shallow water because the nonlinearities that produce extreme wave crests are stronger in shallow water. Measurements in shallow water thus give better tests of both instruments and theories.

The measurements were made during the winter of 1997–1998 from the Meetpost Noordwijk in the southern North Sea. An extensive analysis phase of the project was conducted in 1998–2000. The purpose of this report is to give an overview of the project and introduce the other papers which are presented in this conference.

The next section gives a review of the instrumentation used in the project. The instrumentation included two video cameras which were digitally recorded during all daylight hours. Sea state parameters were calculated for all of the data. 100 hours of the most interesting data were extracted for detailed processing. These data sets were re-sampled to common sampling intervals and the tidal signal was removed. The main purpose of the experiment was to study crest heights. The statistics of crest heights derived from the platform instruments are discussed and we conclude that the waves are distinctly nonlinear. Second order random wave theory can produce simulations which agree very well with the measurements from the most trustworthy of the instruments. Statistics of the wave crests can be derived either directly from the simulations or from a spectral response surface. The final sections discuss some other uses which were made of the data, namely an analysis of crest and trough shapes and the joint height/period distribution.

Instrumentation

The project was set up on the Meetpost Noordwijk (MPN) measurement platform shown in Fig. 1. The platform is a piled steel jacket structure in 18 meters deep water. It is located nine kilometers off the Dutch coast, near the coastal resort of Noordwijk, whence it got its name.
The platform is one of the stations of the North Sea Monitoring Network (Meetnet Noordzee, MNZ), that gathers on-line hydrological and meteorological information from the North Sea. The use of the platform for WACSIS was arranged with the enthusiastic cooperation of the Rijkswaterstaat North Sea Directorate. The Oceanographic Company of the Netherlands (OCN) was the prime contractor for the measurements. A complete description of the instrumentation is given by van Unen et al. (1998).

Wave crest sensors involved were a Baylor Wave Staff, THORN Wave Height Sensor (an upgrade of the EMI Laser), MAREX S05 Wave Radar, SAAB Radar, Vlissingen Step Gauge and Marine 300 Step Gauge. A Directional Waverider Buoy, a SMART 800 GPS Buoy and a WAVEC Directional Buoy were deployed to obtain directional wave information. For current and pressure information, an S4ADW Current Meter was deployed at ten meters below mean sea level. Relevant operational sensors of the MNZ were used as reference instruments, resulting in hydrological and meteorological data sets for validation and calibration purposes.

To facilitate thorough data analysis and data assessment afterwards and possibly to explain any flaws in the measurements due to the various measurement principles, the wave crest measurements were augmented by digitized video frames twice a second during daylight hours. In this way, any problems with regard to the instruments’ response due to implicit filtering, influences of spray and positioning at the platform’s structure could be studied.

Table 1 lists the sensors which were installed specially for the WACSIS JIP and Table 2 lists those which are a normal part of the MNZ. Figure 2 gives a plan view of the position of the sensors on the platform. The EMI laser, Marex radar and Baylor wave staff were located close together while the Saab and step gauges were some distance away on the corners of the platform. The buoys were moored about 1 km to the north of the platform.

The whole data acquisition project for the 1997/1998 storm season effectively started mid December 1997 and ended mid May 1998. A wide variety of conditions were measured, including very stormy periods in early January, mid-January and early March 1998.

### Video Recordings

Video recordings of the sea surface were made throughout the measurement program. The purpose of these recordings was to provide a “reality check” for the wave sensors, and in particular to see whether spray from whitecaps or waves breaking against platform members affected the measurements made by the sensors. No attempt was made to quantify the images recorded by the cameras.

Two video cameras (JAI 1021 monochrome industrial CCD cameras) were installed on the outer corners of the platform. Since most of the wave height sensors were located on the north side and northwest corner of the platform, one of the two cameras was positioned on the northeast corner looking towards the west (along the north side of the platform). The other camera was positioned on the southwest corner looking towards the north (along the west side of the platform). Both cameras were positioned about 1.5 m below the main deck level of the platform, so a clear and free view to the sea surface was guaranteed. The platform structure itself was partly visible in view angle of both cameras to give some points as a reference.
The video cameras were connected to two separate video frame grabbers. The video frame grabbers were contained in two separate Windows NT 4.0 workstations without any further synchronization. The original pictures consisting of 768 by 576 pixels were decimated by a factor two both horizontally and vertically by hardware operations performed inside the frame grabbers. A JPEG image software compression program was then applied for a compression factor of 80% without any visual loss of information for the human eye. During daylight the frame grabbers operated at a fixed rate of two images per second. At that rate, 7,200 JPEG images of approximately 15 kb per hour per camera, i.e. approximately 100 Mb per hour per camera, were recorded on removable hard disks.

Data Selection and Processing

From the five months of nearly continuous data, approximately 100 hours were selected for detailed processing. That data included 50 hrs in daylight, and for those hours digital video (AVI) animations were prepared which included images from both video cameras and a graph of the signals from the wave sensors. An example of one of these frames is shown in Fig. 3.

The 50 hrs of data including video images were supplemented with all of the hours with $H_s > 4$ m, and hours with directions and periods not well represented in the data already selected. These included most of the hours with $H_s > 3$ m. Finally, 20 hrs of waves with low steepness were added for comparison. The hours of data were separated into 20 min files. This data is referred to as the WACSIS Common Data Base. Since there were some sensor malfunctions, not all of the data segments contain data from all sensors.

The change in tide height or storm surge during an hour, although small, can affect zero crossing wave parameters. Therefore, instead of simply subtracting the mean from each record, a cubic spline fit to the 5 min average water level was subtracted.

As noted in Table 1, all of the sensors were not recorded at the same sampling rate. In order to have records from different sensors with common sampling frequencies, the original data was resampled at 4 Hz, 2 Hz, and 1 Hz. This re-sampling was accomplished by Fourier transforming the original time series. Under sampling was done by eliminating frequency components and over sampling was accomplished by adding frequencies with zero amplitude before taking the inverse Fourier transform. Directional spectra were estimated from the data provided by the directional Waverider through calculations of the cross spectra of heave, tilt and roll as described by Kuik et al. (1988).

Sea-State Parameters

Barstow (2002) made extensive calculations of both integral parameters such as significant wave height and individual wave parameters for the WACSIS Common Data Base. The first step of the processing was data control which identified flat spots and spikes in the signals. Spikes are defined as values greater than 4.5 times the standard deviation of the signal. The Marex radar had the greatest number of spikes, 86 in 319 20 min data segments. It was followed by the Saab, Vlissingen, Baylor and EMI laser. The directional Waverider had only two spikes.

The significant wave height is highly correlated between the sensors. There are biases on the order of 2-3% due to calibration errors between the sensors. The standard deviation between the values of $H_s$ increases with distance between the sensors as expected due to the dispersion of directionally spread random waves. The peak periods are similarly well correlated, but with larger standard deviations.

The skewness of the wave records is a measure of the asymmetry.
try of the waves due to nonlinear processes. All of the sensors except for the Waverider showed significant positive skewness as expected, but the amount of skewness differs somewhat between the sensors. The skewness from the Marex radar showed a significant amount of scatter due to the spikes in the signal. For unknown reasons, most of these spikes occurred during the night. Therefore, we compared the skewness recorded during daylight hours, and also restricted the data sets to Hs \(> 1.2 \text{ m} \). The results for 96 simultaneous 20 min records are shown in Table 3.

The Marex and EMI have almost identical and rather low skewness, while the Baylor and Saab are significantly higher. The horizontal skewness of the waves can be conveniently studied by calculating the skewness of the Hilbert transformed time series. The Hilbert transform simply shifts the phase angles of all Fourier components of the sea surface by 90 deg. All of the platform instruments show a very slight tendency for steeper wave crest backs than fronts.

### Table 3 Skewness statistics for simultaneous daytime records for four platform instruments with low sea-state records removed

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Mean</th>
<th>Median</th>
<th>St. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marex Radar</td>
<td>0.147</td>
<td>0.137</td>
<td>0.076</td>
</tr>
<tr>
<td>EMI Laser</td>
<td>0.144</td>
<td>0.139</td>
<td>0.073</td>
</tr>
<tr>
<td>Baylor WS</td>
<td>0.172</td>
<td>0.165</td>
<td>0.072</td>
</tr>
<tr>
<td>Saab Radar</td>
<td>0.199</td>
<td>0.200</td>
<td>0.072</td>
</tr>
</tbody>
</table>

**Interpretation of the Video Images**

Barstow (2002) also conducted a thorough comparison of the video images with the signals recorded by the wave sensors. Typically four or five individual high crests were identified from each 20 min segment for the video analysis. Video sequences at the time of large crests were examined. The main reason for studying the video images was to see if the measurements were consistent with the visual evidence, and, in particular, whether spray affected any of the sensors. The observations fell into three categories:

- Measurements that seem to be consistent with the observations, even though a spike was detected on one or more of the instruments. In these cases, the differences between the instruments could be explained by the spatial variability of the sea surface.
- Measurements of high crests which are consistent between the instruments even though a lot of spray was present. These events give some confidence in the measurements of crests using different instruments.
- Measurements where there is evidence of the influence of spray on the sensors. With the exception of the EMI laser, all of the platform instruments recorded a few anomalously high crests when spray was present. The Saab radar is located on the north-west corner of the platform somewhat away from the other sensors (Fig. 2), and it is interesting to note that almost all of the spurious Saab waves break before reaching the platform or break at the platform leg.

The evidence for elevated crests caused by spray effects is only circumstantial. In many cases, it is possible that wave dispersion or short-crestedness causes the differences between the measurements. It is also impossible to determine what the instruments are actually measuring since we did not actually measure the properties of the surface, but only its appearance. The question is whether the instruments are seeing spray or green water which is dense enough to have a significant effect on a structure. It is possible that careful laboratory experiments of different sensors under controlled conditions could settle these questions.

### Comparisons of Crest Height Measurements

Prevosto and Forristall (2002) made an extensive study of the crest height measurements from the platform sensors. Before looking at some of those comparisons, however, it is useful to examine the wave spectra. Figure 4 shows a representative sample measured at 1400 on 13 April 1998, plotted with log-log scales.

The spectra from the various sensors are very similar near the peak and through the energy containing frequencies, but differ considerably at high frequencies. The black line in the figures has a slope of \(-4\). There remains some debate whether the high frequency tail of the gravity wave spectrum behaves as \(f^{-4}\) or \(f^{-5}\) (e.g., Banner, 1990), but there is no doubt that the true wave spectrum does not curve above \(f^{-1}\) at high frequencies.

The Marex and Saab radars have the highest noise level, generally followed by the Vlissing, Baylor, and EMI gauges. The noise levels stay roughly the same while the energy containing part of the spectrum decreases with decreasing significant wave height. The excess energy there must be due to some form of electronic noise, possibly exaggerated by aliasing from frequencies above the Nyquist frequency. All of the sensors were recorded as either the digital signal provided by the sensor or by directly digitising analog signal from the sensor without any further filtering. It is likely that the sensors provide some form of internal filtering, but the details are not known to us. In any case, the measurements above about 0.5 Hz appear to be unreliable.

Comparison of crest height statistics from the sensors for individual sea states is difficult due to the different locations of the sensors and to the great variability in time and space of the wave field in wind sea conditions. By combining several hours of data with similar wave heights we can eliminate some of the sampling variability in single data records while still being able to see variations due to different ranges of wave heights. While other factors such as wave steepness and directional spreading could also affect the crest height statistics, the range of those variables was apparently not large enough to noticeably affect the statistics.

If the waves could be described by linear theory, the crest heights of zero-crossing waves would follow the Rayleigh distribution given by:

\[
P(\eta > \eta) = \exp \left(- \frac{\eta^2}{\bar{\eta} \cdot m_0} \right) \quad (1)
\]

where \(\eta\) is the crest height, \(H_s = 4 \cdot \sqrt{\eta} \cdot m_0\) is the significant wave height and \(m_0\) is the variance of the wave spectrum. We can get a clear picture of the effect of the nonlinearities on the crest heights...
by normalizing the measured crest height at a given probability level by the height predicted by the Rayleigh distribution at the same probability level. That ratio is then plotted against the probability that the measured crest height was exceeded. Since the crest height ratios are normalized by the significant wave height of the sensor during each record, there are no difficulties with combining records with different significant wave heights or with different sensors measuring slightly different significant wave heights during a sea state.

Figure 5 shows an example of the crest height ratios for the measurements with significant wave heights between 4.0 and 4.5 m. For most of the sensors, 17 hrs of data were in this range. Generally speaking, the Vlissingen gauge showed the highest crests and the EMI laser showed the lowest. The Marex radar recorded a few crests which were very high, to the extent that they are off the scale of the graph. These very high crests, which we believe to be noise of some kind, are not seen when the significant wave height is less than 3.0 m. The Baylor wave staff showed crests which were a few percent higher than the EMI.

Similar figures for other wave height ranges showed very clearly that the crest height ratio increases with increasing significant wave height. This fact illustrates the fundamental nonlinearity of the heights of wave crests. In theory, many other factors such as shape of the spectrum and the degree of directional spreading could influence the crest height ratio, but for this data set, we found that the range of those variables was small, and the crest height ratios could be ranked well by the significant wave height.

Second-Order Simulations of Wave Crests

Forristall (2000) and Prevosto et al. (2000) have independently developed crest height distributions by simulating long time series of wave surfaces which are correct to second order based on the perturbation expansion by Sharma and Dean (1979). Prevosto and Forristall (2002) made a detailed comparison of such simulations with the WACSIS measurements, which we briefly review here.

The simulations begin with the measured spectrum. The amplitudes and phases of the wavelets which make up the spectrum are selected from Rayleigh and uniform distributions, and the time series is calculated through an inverse Fourier transform to give a first order sea surface. The second order terms are then calculated for each pair of wavelets in the first order surface and summed to make the second order simulation. Both uni-directional and directionally spread waves were simulated. For the directional simulations, we modelled the directional spreading with a cosine 2S function.

Tests showed that the two simulation programs gave exactly the same results when the same input parameters were used, but choices in how the simulations are set up can influence the results. The second order perturbation equations perform rather poorly for the interactions between very short and long waves. The simplest way to deal with this problem is to truncate the spectrum at very low and high frequencies. We performed tests by truncating between 1% and 10% of the energy at the high and low tails of the spectrum. The 1% truncation increased the crest heights by 2%–5% for the uni-directional simulations and marginally for the directionally spread simulations. Further truncations reduced the crest heights in both cases. The simulations without truncation resulted in waves with unphysical shapes. Truncating 1% of the energy removes a small part of the nonlinear interactions but avoids the relatively large error in convergence and thus was used in the simulations described in the rest of this report.

To reduce the computation time in the directionally spread simulations, we also truncated directional components which had very low energy. Removing 1% of the directional energy produced almost no change in the simulations. The discretization of the directional spectrum also has an effect. Tests showed that using 12 deg directional sectors was a good compromise between efficiency and accuracy.

Finally, there is the question of which input spectrum should be used. The spectrum from the Waverider includes only part of the second-order bound waves while the platform based instruments measure them reasonably well. Tests however showed that treating the bound waves from the EMI laser or Baylor wave staff as free waves in input to the simulations produced crest height distributions that were essentially the same as those that came from using the Waverider as input.

Prevosto and Forristall (2002) show numerous comparisons of simulations with the measurements. Sampling variability makes it difficult to draw conclusions from single hours of data. Simulations using second order propagation between the locations of the sensors showed that much, but not all, of the differences in crest heights between sensors could be explained by dispersion and directional spreading.

As noted above, all of the measurements appear to be more or less noisy at high frequencies, so the most useful comparisons were made after the measurements were filtered at 0.64 Hz. Measurements with similar values of significant wave height were grouped together to increase the statistical stability of the results, and the theoretical statistics were taken from 1000 hrs of simulations. Parameters other than wave height, such as wave direction and peak period, did not seem to have much effect on the crest height ratios, possibly because these parameters fell within fairly narrow ranges.

Figure 6 shows a comparison of the simulated and measured crest height ratios for significant wave heights between 4.0 and 4.5 m. The filtered measurements divide into two groups, with the crests from the Saab and Marex radars being slightly higher than those from the Baylor, EMI and Vlissingen instruments. The effect of the filtering can be judged by comparison with Fig. 5. The Baylor, EMI and Vlissingen measurements agree well with the simulations while those from the radars are a few percent higher. The three dimensional (directionally spread) simulations are slightly higher than the 2-D simulations for these conditions. For lower wave heights, the filtered measurements are grouped more tightly together with a mean just slightly higher than the simulations.

The comparisons demonstrate that simulations of the wave elevation with a second order irregular 3-D model give statistics of crest height similar to three of the sensors when we consider only wave components up to 0.64 Hz. The power spectra from the sensors indicate that the measurements are very likely to be contaminated by noise above this frequency, so a comparison using the low passed data is reasonable.

The Saab and Marex radar gauges gave crest heights somewhat
higher than the simulations even after the measurements were filtered. It is possible that part of the reason for this difference is a combination of the hydrodynamics of short waves riding on long waves and the way that radar sensors respond to the short waves. The conventional perturbation expansion which we have used for our simulations is not accurate when high frequency waves are included. Thus a simulation based on the phase modulation technique would probably give slightly higher crests than our simulations if high frequency waves were included.

Since the high frequencies were poorly measured they have been filtered out of our comparisons. It is possible, however, that the some effect of the high frequency waves remains in the radar measurements due to their relatively large footprints. If a radar senses the highest point in its footprint and if the wavelength of the short waves is comparable to the size of the footprint, then the high frequency waves will be aliased into lower frequencies and not removed by the filter. We have not been able to confirm that the radars actually do sense the highest point in their footprints, so this explanation is still speculative.

Crest Statistics from a Spectral Response Surface

While the second order simulations give accurate crest heights, they are too time consuming for regular engineering use. Prevosto and Forristall (2002) have therefore proposed two types of parametric models which fit their simulations. These models will be discussed in the concluding section. Tromans (2002) has attacked the problem more directly by working in the probability domain. The calculations are an application of first-order second moment reliability methods (FORM).

As in the simulations, Tromans begins with an underlying linear process defined by a directional wave spectrum. The linear process is modified by the second order interactions between its components. The first plus second order surface elevation can then be written in the form:

\[ \eta = \sum_j \sigma_j x_j + \sum_{jk} F_{jk} \sigma_j \sigma_k x_j x_k + H_{jk} \sigma_j \sigma_k \tilde{x}_j \tilde{x}_k \]  

(2)

where \( x_j \) is the jth linear wave component normalized by its standard deviation \( \sigma_j \), and \( F_{jk} \) and \( H_{jk} \) are the second order interaction kernels modified to operate on \( x_j \) and its Hilbert transform \( \tilde{x}_j \). Since \( \{x_j, \tilde{x}_j\} \) are jointly normal with zero cross correlation, surfaces of constant probability density are hyperspheres. The point on the surface where \( \eta \) is a maximum is found using Lagrange's method of undetermined multipliers and a variation on the iterative scheme suggested by Hasofer and Lind (1974).

The method was tested by comparing the results with the time domain simulations described by Forristall (2000) and the agreement was very good. Since the calculation time for the spectral response surface method is only a few seconds, it appears to be a viable alternative to the parametric models given by Prevosto and Forristall (2002).

Crest and Trough Shapes

Taylor and Williams (2002) investigated the shape of the waves by extracting short segments of the records around crests and troughs. They then aligned the maxima in each segment and averaged the time series. The averages were formed for bins with various peak amplitudes. According to NewWave theory (Tromans et al., 1991), the shapes of the high waves should be similar for a few seconds around the crest, at least to first order. Results were plotted for both in dimensional form and non-dimensionalized by the peak amplitude of the averaged record.

The resulting average profiles show a clear vertical asymmetry. For the larger waves, the crest amplitudes are typically about 20% larger than the troughs. The crests are sharper than the troughs, and the average crest shape for the Marex and Saab sensors are somewhat sharper than the average crest shape from the Baylor wave staff. The horizontal asymmetry of the waves was investigated by taking the Hilbert transform of the time series. Some slight asymmetry was found for the largest waves, but it was thought to be negligible.

To second order, the shape of the underlying linear wave can be found from:

\[ \eta_{\text{lin}} = \frac{\eta_{\text{crest}} + \eta_{\text{trough}}}{2} \]  

(3)

and the shape of the second-order terms from

\[ \eta_{\text{2nd}} = \frac{\eta_{\text{crest}} - \eta_{\text{trough}}}{2} \]  

(4)

Figure 7 shows the resulting average linear and second order wave shapes for the storm of Jan. 6, 1998. This example uses data from

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**Fig. 6** Comparison of crest height ratios of measurements low pass filtered at 0.64 Hz and second order simulations for significant wave heights between 4.0 and 4.5 m.

**Fig. 7** Average linearized ad second order shapes of waves measured by the Saab radar during the storm of Jan. 6, 1998. The legend indicates the amplitude bins over which the averages were taken.
the Saab wave radar. The different lines in the figure show the result of averaging over waves of various amplitudes, as identified in the legend. For example the curve labeled 0%–2.5% means the average was taken over the highest 2.5% of the waves.

The linear wave shape is almost identical regardless of wave height for four or five seconds around the crest. These linear wave shapes agree very well with the NewWave model proposed by Tromans et al. (1991) which predicts that the most probable shape of a high wave is given by the auto-correlation function of the wave signal, conveniently calculated as the Fourier transform of the wave spectrum. The measurements have just slightly sharper crests than the calculated NewWave shapes (not shown here). The discrepancy may be due to the fact that the linearization of equation (3) does not remove the odd numbered nonlinear terms from the signals, but it may also be due to noise in the measurements. As noted above, the radar sensors reported sharper peaks than the Baylor wave staff or EMI laser.

As expected, the second order signal in Fig. 7 has approximately twice the frequency of the linear shape. The second order terms raise both the crest and trough of the first order signal, sharpening the crest and flattening the troughs.

Taylor and Williams (2002) plotted the amplitude of the second order shape against the steepness of the first order waves, and found

\[ a_2 \approx 0.4k a_1^2 \]

where \( a_1 \) is the first-order amplitude and \( a_2 \) is the second-order amplitude, for a wide range of wave steepness. This is as we would expect for a narrow band process.

### Wave Height and Period Distribution

The WACSIS project included an analysis of the joint wave height and period distribution. Joint statistics from two 40 minute recordings, which agree very well with the NewWave model proposed by Cavanie et al. (1991) which predicts the most probable shape of a high wave is given by the auto-correlation function of the wave signal, conveniently calculated as the Fourier transform of the wave spectrum. The measurements have just slightly sharper crests than the calculated NewWave shapes (not shown here). The discrepancy may be due to the fact that the linearization of equation (3) does not remove the odd numbered nonlinear terms from the signals, but it may also be due to noise in the measurements. As noted above, the radar sensors reported sharper peaks than the Baylor wave staff or EMI laser.

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**Application to the Calculation of Maximum Crest Heights**

While the second-order simulations appear to give a good replica of natural waves, it would be awkward and time consuming to calculate such simulations for routine engineering design. Prevosto and Forristall (2002) have thus developed parametric models which closely match the results of their simulations. The narrowband model is based on the assumption that the wave spectrum is sufficiently narrow so that the second order transfer functions can be considered to be constant. This assumption leads to a formula that relates the second order crest height \( C \) to the linear crest height \( C_{lin} \) by:

\[
C = C_{lin} + \left( T_{1/2}^D(f_m) + T_{1/2}^S(f_m) \right) C_{lin} - T_{1/2}^D(f_m) \frac{H_s^2}{8} \tag{5}
\]

where \( T_{1/2}^D \) and \( T_{1/2}^S \) represent the second order transfer function for the difference and sum terms, evaluated at the peak frequency \( f_m \). The significant wave height \( H_s \) and \( f_m \) are modified empirically to account for spectral bandwidth and directional spreading.

Prevosto and Forristall (2002) gave another parameterization of their simulations, in which Weibull distributions of the form

\[
P(C) = \exp \left( -\frac{C}{\alpha H_s^\beta} \right) \tag{6}
\]

where \( \alpha \) and \( \beta \) are functions of the wave steepness and Ursell number \( U = H_s k^2 / d^2 \).

One would often like to use the distribution of crest heights to estimate the maximum crest height in an hour, a storm, or 100 years. Krogstad (2002) has considered this problem using the WACSIS data for verification.

The engineering approximation for the calculation of the maximum crest while the sea state is stationary assumes that all of the individual waves are independent. Asymptotic Gaussian theory suggests however that regardless of the spectral shape, it is correct to assume that crest heights are Rayleigh distributed and raise the distribution to the power \( N \) of zero-crossing waves to determine the maximum crest. The result is the same asymptotic Gumbel distribution as is derived in Leadbetter et al. (1983) for Gaussian processes under very general conditions.

With the Gaussian result as a guide, Krogstad (2002) calculated the maximum crest heights in the 17 min WACSIS records by raising equations (5) and (6) to the power of the number of zero-crossing waves. He found that the results agreed quite closely with the maximum crests measured by the EMI and Baylor sensors, while the other three sensors did not agree with the calculations. This result is consistent with the comparisons of crest height distributions such as those shown in Fig. 6. The Marex data of course includes many large noise spikes. It should also be noted that the data used by Krogstad was the unfiltered 4 Hz data, and the filtering had the most effect on the Marex and Saab data as is evident from Fig. 4.
Krogstad also showed how the theory could be applied to calculate the distribution of the maximum crest height during a storm. He used the time series of significant wave height and period for a storm at Frigg field in the North Sea as an example. The maximum significant wave height during the storm was about 12.5 m. To illustrate the effect of water depth on crest heights, he varied the depth from infinitely deep to 70 to 25 m. The results are shown in Fig. 9. The curves show the probability density of the maximum crest during the storm. The change from infinite depth to 70 m is marginal, but if the storm had occurred in the rather unrealistic depth of 25 m the change in crest height would have been considerable.

Conclusions

The WACSIS project collected a very interesting data set which has enabled us to give reasonably definite answers to the questions which were posed at the start of the experiment. The most important questions concerned the accuracy of the various sensors which can be used to measure wave crest elevations and the verification of one or more theoretical models for the crest height distribution.

We cannot really determine the accuracy of the sensors since there is no absolute standard against which they can be judged. The most we can hope for in a comparison experiment is that a consensus will develop from which a reasonable judgment of the physical truth can be made. In that sense, we were successful since the EMI laser, Baylor wave staff and Vlissingen step gauge gave very similar results after the data were filtered at 0.64 Hz. The filtering is reasonable since the raw data have unexpectedly high spectral densities above this frequency. The Saab radar gave very similar results, both of which are very close to the results of the simulations and thus very close to the best measurements. Krogstad (2002) has given a more formal verification of equation (6) by using a Kolmogorov-Smirnov test applied to the maximum crest heights discussed in the previous section. The test accepts the second order model in comparisons with the EMI and Baylor data for which we have the most confidence. Given these results, we are confident in recommending the use of Equations (5) or (6) in engineering calculations of wave crest heights.

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