Climatic wave spectra and freak waves probability

Alexander V. Boukhanovsky[1,2], Leonid J. Lopatoukhin[1], Carlos Guedes Soares[2]

1 – St. Petersburg State University, St. Petersburg, Russia
2 – Unit of Marine Technology and Engineering, Technical University of Lisbon, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisbon, Portugal
avb@csa.ru, leonid@LL1587.spb.edu, guedess@mar.ist.utl.pt

Abstract. Long-term ensembles of hindcasted sea wave spectra are analyzed in order to provide a statistical description of spectral wave climate. The possibility to identify situations of high probability of freak wave occurrence is considered by analyzing the situations in which there has been rapid transitions between types of wave spectra as classified in the climatic spectra evaluation.

1 Introduction

Nowadays the main source of wave climate information is based on the results of hydrodynamic simulation of wave generation by past wind fields (in other words hindcasting). Spectral models, mainly WAM, Wavewatch and SWAN are being widely used. The result of hindcasting is a set of wave fields described by directional spectra at selected grid points. Normally the statistics that are determined, based on the moments of directional wave spectra, are the significant wave height h_s, mean period $\bar{\tau}$, and the mean wave direction $\bar{\theta}$. These parameters contain the general information about the intensity of the waves, but they do not describe the spectral structure of complex sea states so completely as the spectra do.

The first attempt to analyze the long-term variability of sea wave spectra was made by Scott [14]. The average wave spectra were computed using 204 wave records at OWS "I" in the North Atlantic. *Buckley* [1] analyzed more than 2 million spectra that were measured during 12 years at 13 buoys located in coastal waters of the USA. All the wave situations were subdivided down into twelve types, based on the values of significant wave height.

The main methodological approach to provide average spectra that can be considered a spectral wave climate description is connected with principles of "averaging" of both the frequency and directional spectra, for which there are many different techniques. For instance, *Vincent & Resio* [16] represent the spectral ensemble using the expansion with respect to empirical orthogonal functions, but the interpretation of the principal components sometimes is not clear. *Ochi* [12] analyzed 800 spectra and showed that the most general representation would be the composition of one-peaked spectra with the simple set of parameters. Statistical models of long-term variability of multi-peaked spectra were considered by *Guedes Soares et al.* [4,7] and *Lopatoukhin et al.* [8,9] for omni-directional case.

The principal goal of this paper is the development of a parametrical statistical model for climatic multi-peaked directional wave spectra, and to suggest the possibility to use these results for estimation of freak wave probability arising [11]. Freak (or rogue) waves are one of the main hazards at sea. Most existing freak wave measurements are recorded in some point of the sea, but most short-term statistics do not show such phenomena. The directional spectrum of wave record does not reveal the existence of freak wave and consequently the results of hindcasting for any specific time also do not display any suspicion of such type of waves. However synthesis of hindcasted spectra, i.e. their long-term (climate) features opens the perspective to identify situations of higher probability of occurrence of the freak wave phenomena as will be shown in the last section here.

2 Initial dataset and problem to solve

Statistical models of directional wave spectra may be developed for any sea or part of the Ocean. The North Sea was selected for this study as being one of the most investigated areas, where many wave measurements and data are available. The information about long-term distributions of spectral parameters are known and published.
The data used here is the continuous ensemble of the hindcasted sea wave fields from 1983 till 1998 (each 3 hours), obtained with the help of Wavewatch III (version 2.22) model for a grid of 15×15 miles. The NCEP/NCAR reanalysis wind speed (at 10-m level) was used as input. In each point the directional sea wave spectrum $S(\omega_i, \theta_j)$ has 24 values in direction θ_j (step 15^0) and 25 in frequency ω_i. The total number of the sea grid points is more 1500. Thus, the integral size of the output data (arrays of the directional wave spectra) is enormous. Total number of spectra is more than 65 millions, and the bulk of data is $4 \cdot 10^{10}$ numbers.
The main problem to solve is the statistical generalization of such data, taking to account the physical features of sea waves and specifics of the data representation. High dimension of these values and the complexity in interpretation induces one to adopt some simplifications in calculations of wave climate statistics and in particular in the form of spectra. This simplification is based on the parameterization of the each directional spectrum

$$S(\omega,\theta) = S\big(\omega,\theta \mid \Xi(\vec{r},t)\big), \tag{1}$$

which is defined as a deterministic function of random arguments $\Xi(\vec{r},t)$. Thus, the formulation (1) allows reducing all the results of statistical modeling to a space of spectral parameters.
There are some challenges in the achievement of a clear result in statistical modeling of directional wave spectra. A *methodological* challenge is caused by the fact, that the traditional techniques for statistical formalization of sea wave spectra (e.g. principal component analysis, clustering without learning etc.) are not adequate to problem solving, because they not take into account both the physical features of sea waves

and data specifics. Therefore, the alternative statistical approaches highlighting the physics of $\Xi(\vec{r}, t)$ are needed.

A *performance* challenge reflects the strong requirement to computational procedure due to high amount of data. In spite of these challenges, the formulation (1) allows one to achieve the main objectives of spectral wave climate investigations:

- to select classes of wave spectra and to estimate their probability;
- to propose a parameterization which allows one to justify a choice of difference between the various classes (i.e., the selection of discriminant variables);
- to approximate the ensemble $\{S(\omega,\theta)\}$ in terms of its probabilistic characteristics;
- to elaborate a stochastic model of the spectral wave climate;
- to analyze the association between climatic wave spectra and rare events (the possibility of freak waves).

The problems of the classification and the discriminant analysis of climatic wave spectra were considered in [10]. Here only the problems of statistical modeling of spatio-temporal variability and approximation of the statistical characteristics are considered. Using of climatic wave spectra as the base of freak wave probability forecasting is discussed.

3 Statistical parameterization of directional spectra

In the present study, parameters of the spectrum related to wave height, spectral shape, the frequency of the spectral peak, ω_{max}, and the main wave direction, θ_{max}, are selected as parameters in Ξ. The single peaked model spectrum may be written as $S(\omega/\omega_{max}, \theta-\theta_{max}, \Xi)$, where Ξ signifies the set of the spectral parameters.

The more general multi-peaked spectrum $S(\omega,\theta)$ are obtained as

$$S(\omega,\theta) = m_{00} \sum_{p=1}^{n_{fields}} \kappa_p S_p(\omega,\theta \mid \omega_{max}^{(p)}, \theta_{max}^{(p)}), \tag{2}$$

where m_{00}, the 0th moment of the spectrum, is equal to the total variance of wave field, n_{fields} is the number of wave fields (peaks in the spectrum), and κ_p are weight factors for each system so that $\sum_{p=1}^{n_{fields}} \kappa_p = 1$.

Guedes Soares [3] has proposed that two-peaked spectra could be represented by a combination of two JONSWAP components, which was also adopted by Torsethaugen [15] and more recently confirmed by Ewans et al [2]. However, in view of the heavy computational demand, a simpler frequency model spectrum - the Gamma-spectrum – was adopted both for wind sea and swell:,

$$S_\Gamma(\omega,\omega_{max},n) = n/\omega_{max} \left(\omega/\omega_{max} \right)^{-n} \exp\left(-\left(\frac{n}{n-1} \right) \left(\frac{\omega}{\omega_{max}} \right)^{1-n} \right). \tag{3}$$

Note, that $\int_0^\infty S_\Gamma(\omega,\omega_{max},n) d\omega = 1$, and that the spectral peak occurs for $\omega=\omega_{max}$. For the directional distribution, a well known one was adopted with the form:

$$Q_0(\theta,\theta_{\max},m) = C_m \cos^m(\theta-\theta_{\max}), \quad |\theta-\theta_{\max}| < \pi/2, \tag{4}$$

where C_m is a normalizing parameter such that $\int_0^{2\pi} Q_0(\theta,\theta_{\max},m)d\theta = 1$, and the m parameter determines the width of the angular distribution.

The model of one-peaked directional spectrum used below is therefore

$$S_p(\omega,\theta \mid \omega_{\max},\theta_{\max},n,m) = S_\Gamma(\omega,\omega_{\max},n)Q_0(\theta,\theta_{\max},m) \tag{5}$$

This means that the building blocks of S_p are defined in terms of four parameters ω_{max}, θ_{max}, n, and m. The full spectrum shown in (2) is completely defined by specifying the overall energy in the waves, m_{00}, the weights κ_p, and the parameters for each system, $\{\omega_{\max i}, \theta_{\max i}, n_i, m_i\}$. From these expressions one can derive the marginal spectra, $S(\omega)$ and $Q(\theta)$, defined from the integration of the expression in Eqn. (2) over direction and frequency, respectively.

The parameters m_{00}, $\omega_{\max 1}$, and $\theta_{\max 1}$ are determined directly from the spectra for the main peak. These values define the total energy of the spectrum, and the location and number of the prevailing wave fields.

The secondary wave systems are characterized by parameters $\omega_{\max i}$ and $\theta_{\max i}$, $i = 2, 3, 4...$; but their definition are not obvious everywhere. Lopatoukhin et al [10] have considered the simplified case, where all wave systems ($\omega_{\max i}$,$\theta_{\max i}$) correspond to the clear separate peaks in the directional spectrum. But in fact, there are some situations, where the summary spectrum of wind sea and swell have only one single clear peak with the broad peak corresponding to the input of a secondary wave system. To reveal such cases the parameterization procedure of [11] was modified on the base of sequential conditional optimization technique. The brief algorithm considers the minimization of the functional, called "the deviation index":

$$DI = \sum_i \sum_j \frac{\left| S(\omega_i,\theta_j) - S^*_{ij} \right|}{S^*_{ij} m_{00}} \xrightarrow[\langle \kappa_k, \omega_{\max k}, \theta_{\max k}, N \rangle]{} \min, \tag{6}$$

with the conditions

$$\sum_{p=1}^{n_{fields}} \kappa_p = 1; \quad \left|\theta_{\max i} - \theta_{\max j}\right| \ge \Delta_\theta \vee \left|\omega_{\max i} - \omega_{\max j}\right| \ge \Delta_\omega, \quad i,j = \overline{1,N}. \tag{7}$$

where S^*_{ij} are the values of the model output (for frequency ω_i and direction θ_j). Sensitivity values Δ_θ and Δ_ω are driving the algorithm calibration; in this work the values adopted were $\Delta_\theta = 30^0$ and Δ_ω=0.1 (rad/s). Solving of the problem in equations (6,7) was done by means of adaptive Monte-Carlo approach for n_{fields} = 2, 3,... in sequence. The computations were finished when the $\kappa_{n_{fields}} < \Delta_\kappa$, where Δ_κ is the sensitivity parameter (in this case, Δ_κ=0.05, i.e., the secondary wave system was considered negligible, if its energy input in m_{00} less than 5%).

Thus, any directional spectrum in the ensemble may be parameterized in terms of model (2-5), using the optimization procedure (6,7). But the different number of wave systems n_{fields} in (2) lead to spectra with the principally different shapes that may be described by means of statistical classification.

4 Spatio-temporal statistics of climatic wave spectra

4.1. Occurrence of the spectral classes

The model (2) allows one to distinguish one-, two- and multi-peaked (by variables ω and θ) spectra. Hence, a generic classification may be presented as shown in Fig. 1 for the SW part of the North Sea. The details of the classification procedure are published in [10].

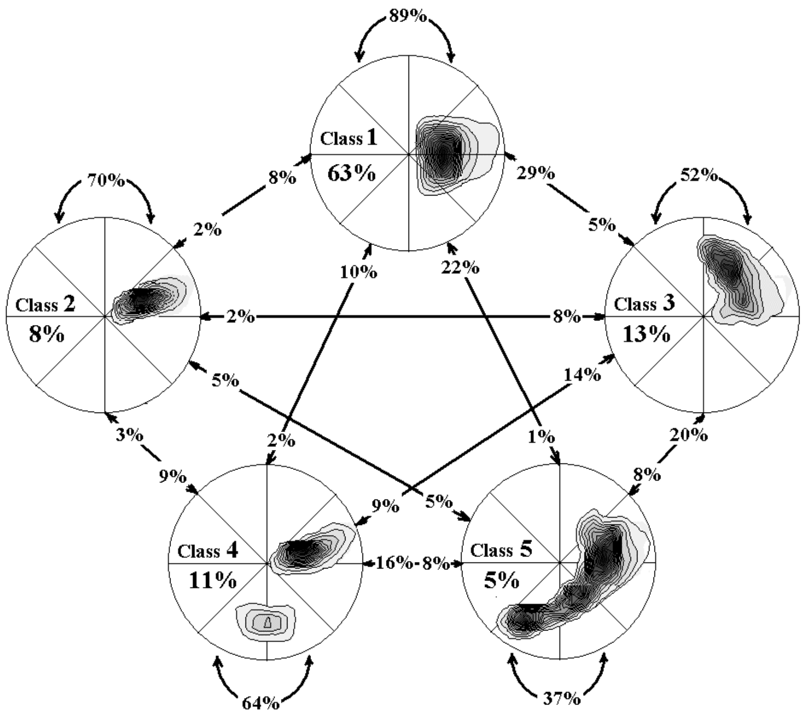


Fig. 1. Classification and transient “star” diagram for directional spectra variability. SW part of North Sea.

The following five classes of wave spectra are selected:

One-peaked spectra (n_{fields} =1)

- Wind waves (k=1);
- Swell (k=2);

The separation between wind waves and swell is based on the non-dimensional steepness defined as

$$\delta = \frac{g\tau_{\max}^2}{h_s} = \frac{\pi^2 g}{\sqrt{m_{00}}\,\omega_{\max}^2}, \qquad (8)$$

If δ>300, then spectrum belongs to a swell, otherwise to wind waves.

Double-peaked spectra (n_{fields} =2)

- Wind waves and "young" swell with close frequencies (k=3); It means, that in (7) may be $\left|\omega_{\max i} - \omega_{\max j}\right| < \Delta_{\omega}$.
- Wind wave and "old" swell separated both by frequency and direction (k=4);

Multi-peaked spectra (n_{fields} ≥3).

- Wind waves and swell without separation (complicate sea) (k=5).

The spatial distribution of occurrences of each class of the spectra in the North Sea is shown in the Fig. 2. It is seen, that the wind waves are prevailing all over the sea. The occurrence of complex sea with "fresh" swell is decreased from North to South.

4.2. Markov transitions probabilities between classes

Associating each class with the stable state of the sea with number k, the synoptic variability of sea waves may be presented as the Markov chain $k = k(t)$ with the transient probability matrix $p_{ij}^{(t,t+1)} = P\{k^{(t+1)} = i \mid k^{(t)} = j\}, \quad i, j = \overline{1,m}$ and limit probability vector $\pi_j = P\{k^{(t)} = j\}, \ j = \overline{1,m}$. In Fig. 1 the transitions between classes are also shown as a "star" diagram, where the arrows correspond to different transient probabilities. E.g., the probability of transition during 3 hours from *Wind waves* (Class 1) to *Wind waves and "young" swell* (Class 3) is 5%, and 29% - return. The probability of the cases with the same class after 3 hours, is pointed on the arcs; e.g. for the *Wind waves* this value is 89%.

4.3. Approximation of spectral statistics

The model (1) allows one to estimate the probability characteristics of directional spectra S(ω,θ) – mean value, r.m.s, probability, tolerant and probability intervals by means of the statistical linearization. The mean spectrum is:

$$\bar{S}(\omega,\theta) = S(\omega,\theta,\bar{\Xi}), \tag{8}$$

the $p\%$ quantile spectrum is:

$$S_p(\omega,\theta) = S(\omega,\theta,\Xi_p), \tag{9}$$

spectral variance is:

$$D_S(\omega,\beta) \cong \sum_{i=1}^{n}\left(\frac{\partial S(\omega,\beta)}{\partial \xi_i}\right)^2_{\xi=\bar{\xi}} D_{\xi_i} + 2\sum_{i>j}\left(\frac{\partial S(\omega,\beta)}{\partial \xi_i}\right)_{\xi=\bar{\xi}}\left(\frac{\partial S(\omega,\beta)}{\partial \xi_j}\right)_{\xi=\bar{\xi}} \operatorname{cov}(\xi_i,\xi_j). \tag{10}$$

Here $\bar{\Xi}, \Xi_p$ – are the sets of mean or quantile parameters of spectra, $D_{\xi_i}, \operatorname{cov}(\xi_i,\xi_j)$ the variance and the covariance of these parameters. For example, in Fig. 3 the results of the estimation of mean spectra with 70% probability intervals for each class, for

SW part of the North Sea, are presented. All the directions are showed in Wavewatch III notation (zero is the East, and rotation counter clockwise).

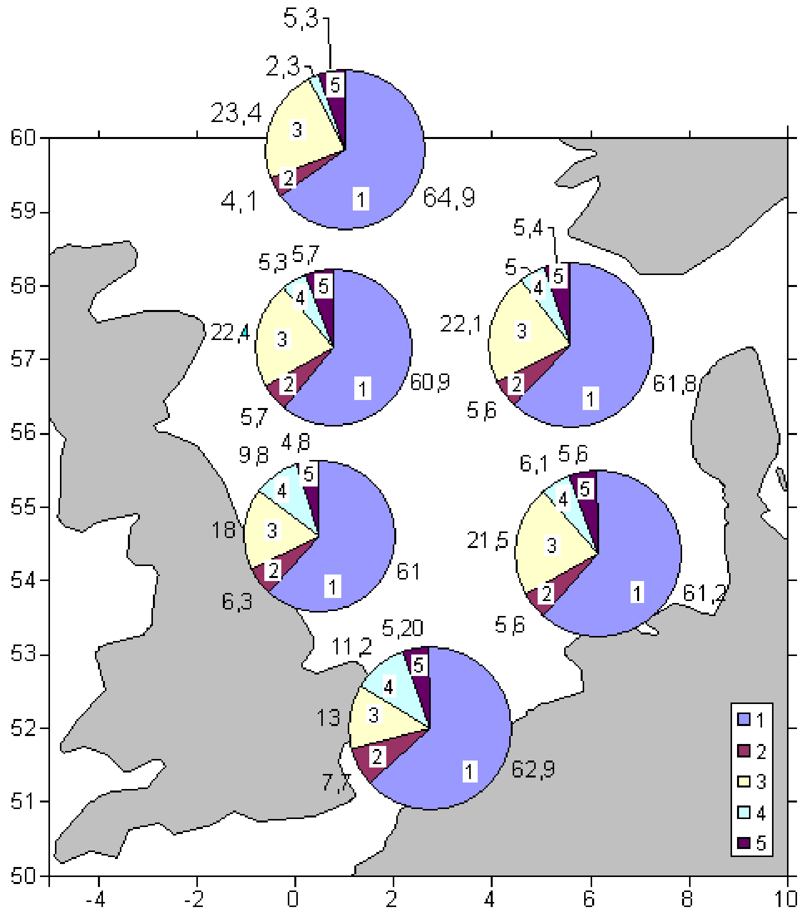


Fig. 2. Spatial distribution of the occurrence of 5 classes of directional spectra in the North Sea. 1÷5 are the classes of spectra. The probability (%) shown near the pies.

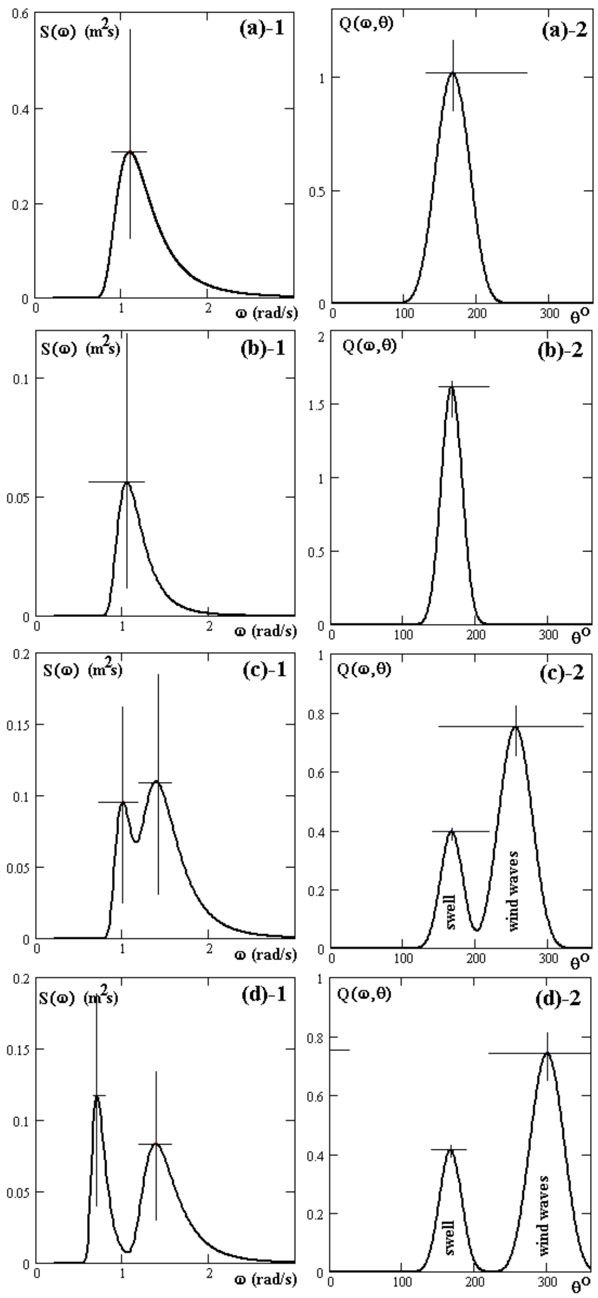


Fig. 3. Mean values and 70% probability interval for frequency spectra and angular distributions of directional spectra in Eq. (2). Classes 1-4, SW part of the North Sea.

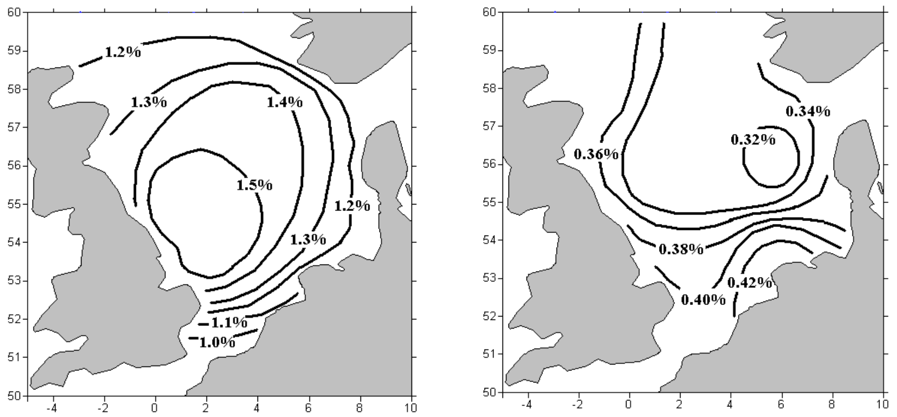
5 Rapid sea state transitions and the occurrence freak wave

At the present state of knowledge the prediction of individual freak wave occurrence seems to be impossible. Nevertheless, the situations with increased freak wave probability may be described [11]. Comprehensive list of external and internal scenarios of freak wave generation are published elsewhere (e.g. [13]).

One of the metocean scenarios leading to freak waves generation are rapid changing of wave conditions. Using the data from climatic wave spectra this is a situation with jump from one class of spectra to another and returning back to initial situation. The most suspicious jumps related to this situation are the ones from class 1 to class 5 and back (with probability $P_{151} = \pi_1 p_{15} p_{51}$), and the jumps from class 1 to class 3 (with probability $P_{131} = \pi_1 p_{13} p_{31}$).

Markov transient probabilities were calculated both for time and space domain. In the time domain the highest probability is on the diagonal of the matrix. This means, that at least during 3 hours, the class of spectra remains the same. The next one in terms of probability is the transition from class 1 (*wind waves*) to class 3 (*wind waves and fresh swell*) is 22-27% for all the regions. The lowest probability is for transition from any class to class 5 (*wind waves and swell without separation*). Such situations are quite rare, but their probability is not infinitely small.

The plots with those probabilities are shown on the Fig. 4. It is seen that the probability P_{131} has a maximum in the West side of the North Sea, and the probability P_{151} increase from the North part of sea to South.



(a) (b)

Fig. 4. Probability of the "spectral jumps" (from wind waves to complicated sea and back): (a) - P_{131}, (b) - P_{151}.

As has been pointed out, the probability of freak wave occurrence increases in the cases of jumping from one class spectra to another. This is confirmed by comparing the present climate results with wave measurement in some points in Northern North Sea.

Cases where freak waves were recorded in the North Sea were analyzed by Guedes Soares et al [5,6] respectively for North Cormoran and for North Alywn and Draupner. These situations have been studied by investigating in the database of hindcast wave spectra the transitions that have occurred between types of spectra at the time of occurrence of the freak waves. The results are presented in Table 1. In the same table (row 3) jumps between different classes of spectra (1÷5) are shown.

Additionally, in this table the instrumental measurements in the Black sea [8,11], where the freak waves were recorded, are considered. The result confirms that in all cases of freak wave recording, there happened sudden change of classes of spectra.

From Fig. 4 it is seen that for case of jump 1-3-1 the most dangerous is the Eastern part of the sea (probability 1.5%). In the case of jump 1-5-1 – the most dangerous is Southern part of the sea (probability 0.4%).

Forecasting of wave conditions for the North Sea is a routine experience. This means, that changing of wave conditions are also known. Therefore prediction of spectral jumps may be one of warning to the possibility of freak wave arising.

Table 1. Dates and types of spectral jumps from the one class to another (in comparison with the wave measurements in the North Sea and the Black Sea).

Position	Date	Sequence of classes (each 3 hours)
North Alwyn, N. Sea	16.11.1993	1 **1 3** 3 **3 1** 1 1
"	18.11.1993	1 1 1 **1 3** 3 **3 1**
N. Cormorant, N. Sea	04.01.1993	**1 5 5 3 1** 1 1 1
"	12.01.1993	3 3 3 **3 1** 1 1 1
"	18.01.1993	**1 3** 3 3 3 **3 1** 1
"	12.03.1996	1 1 1 1 1 1 1 1
Draupner, N. Sea	01.01.1995	1 1 **1 3 3 1** 1 1
Gelendzik, Black Sea	16.12.2000	4 3 **3 1 1 3 1 3**
"	22.11.2001	4 3 3 3 **3 1 3 1**

6 Conclusions

An approach for the statistical modeling of climatic directional wave spectra is proposed. Probabilities of occurrence of each class of spectra, and of the transitions between classes for the North Sea are estimated (see Fig. 1, 2). Jumps from one class of spectrum to another have been studied for the time in which cases of freak wave measurements in the North and Black sea were reported. It is suggested that those types of rapid jumps may be regarded as an indication of increased freak wave probability.

Acknowledgements

This research is partially funded by INTAS grant 2001-2156 “Freak waves generation in the ocean”.

References

1. Buckley W.N. Extreme and climatic wave spectra for use in structural design of ships. *Naval Engineering Journal Soc*. (1988), pp. 36-57.
2. Ewans, K. C.; Bitner-Gregersen, E., and Guedes Soares, C. Estimation of Wind-Sea and Swell Components in a Bimodal Sea State. *Proceedings of the OMAE Specialty Conference on Integrity of Floating Production, Storage & Offloading (FPSO) Systems;* 30 Aug 2-2004 Sep 2; Houston, TX. (2004), Paper OMAE PFSO'04-0045
3. Guedes Soares, C. Representation of Double-Peaked Sea Wave Spectra. *Ocean Engineering*. (1984); Vol. 11, pp 185-207.
4. Guedes Soares C. On the occurrence of double peaked wave spectra, *Ocean Engng., Vol. 18, No 1/2,* (1991), pp. 167-171.
5. Guedes Soares, C.; Cherneva, Z., and Antão, E. M. Characteristics of Abnormal Waves in North Sea Storm Sea States. *Applied Ocean Research.* (2003), Vol. 25(6), pp 337-344.
6. Guedes Soares, C.; Cherneva, Z., and Antão, E. M. Steepness and Asymmetry of the Largest Waves in Storm Sea States. *Ocean Engineering.* (2004), Vol. 31, pp 1147-1167.
7. Guedes Soares, C. and Nolasco, M. C. Spectral Modelling of Sea States with Multiple Wave Systems. *Journal of Offshore Mechanics and Arctic Engineering*. (1992), Vol. 114, pp 278-284.
8. Lopatoukhin L.J., Boukhanovsky A.V., Rozhkov V.A., Divinsky B.V. Climatic wave spectra of the Black Sea, *Proc. Int. MEDCOAST Conf. «Wind and wave climate of the Mediterranean and the Black sea», Antalya, Turkey*, (1999) pp. 97-109.
9. Lopatoukhin L.J., V.A.Rozhkov, V.E.Ryabinin, V.R.Swail, A.V. Boukhanovsky, A.B. Degtyarev. Estimation of extreme wind wave heights. *World Meteorological Organization. WMO/TD–No 1041*, (2000), 76 p
10. Lopatoukhin L., Rozhkov V., Boukhanovsky A., Degtyarev, A., K. Sas’kov, G. Athanassoulis, Ch. Stefanakos, H. Krogstad. The spectral wave climate in the Barents sea. *Proceedings of Int. Conf OMAE’02, Oslo, Norway, June 23-28*, (2002), (CD-version).
11. Lopatoukhin L.J., Boukhanovsky A.V. *Freak waves generation and their probability. Int. Shipbuild. Progress*, 51, 2-3, (2004), pp. 157-171.
12. Ochi M.K. Wave statistics for the design of ships and ocean structures. *Trans. Soc. Naval. Architects and Mar. Eng. Vol.86*, (1978), pp. 47-76.
13. Olagnon, M, Athanassoulis, M., *Proceedings of the Rogue waves Workshop, Ifremer. Brest, France.* (2000).
14. Scott J.R. Some average sea spectra. *Quarterly Transactions Royal Institution if the Naval Architects, v.110, N 2*, (1968) pp. 233-245.
15. Torsethaugen, K. (1993), "A Two Peak Wave Spectrum Model", *Proc. of the 12th Int. Conf. on Offshore Mechanics and Arctic Engineering*, Vol. 2, pp. 175-180.
16. Vincent C.L, Resio D.T.. An eigenfunction parameterization of a time sequence of wave spectra. *Coast. Eng. Vol. 1*, (1977) pp. 185-205.