On the role of downshifting in formation of large wave events

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Fully nonlinear modeling: The long time evolution of long wave packets which split into a number of envelope solitons is studied. The split-up process is highly nonlinear, with locally very high steep waves being formed, even if the initial wave slope is rather small. The initial state of the wave field is specified in two steps. First an exact steady Stokes wave, with wavenumber k_0 and amplitude a_0 (half the total wave height) is computed. Secondly, the surface elevation and the tangential velocity at the surface are multiplied by the 'bell' function $\operatorname{sech}\left[\,\epsilon_0\,\sqrt{2}\,a_0 k_0^2\,(\,x - x_0\,)\,\right]$ where the parameter ϵ_0 determines the length of the wave packet. An exact soliton solution of the nonlinear Schrödinger equation (NLS) is obtained with $\epsilon_0 = 1$. An initial condition with $a_0 k_0 = 0.09$ and $\epsilon_0 = 0.26$ is input to a fully nonlinear model (Clamond and Grue, 2001).

The computational domain involves 128 wavelengths and the carrier wave is discretized over 32 nodes per wavelength. This means that all harmonics up to the 15th are resolved, and that 128 Fourier modes are included in the spectral band $[k_0 - \frac{1}{2}k_0; k_0 + \frac{1}{2}k_0]$. The fully nonlinear simulations show that three large wave events occur during the 3000 wave periods of simulation: number one at $t/T_0 = 155$ wave periods ($k_0\eta_m = 0.2866$, $k\eta_m = 0.34$ - k the local wavenumber - which is the maximal wave elevation observed during the computations), number two at $t/T_0 = 410$ wave periods ($k_0\eta_m = 0.2545$), number three at $t/T_0 = 627$ wave periods ($k_0\eta_m = 0.2704$).

The simulations show that a number of three solitons is formed within the wave envelope, that the ongoing interaction between the two leading solitons form large wave events, and that the smallest soliton in the end of the wave group is the first to detach from the group. (From about $t/T_0 \approx 1200$ the wave field consists of three separated solitary wave groups with ordered heights, the steepest being ahead.)

The spectral content of the wave group may be illustrated by the Fourier transform $\mathcal{F}(\eta)$ (figure 1) and the wavelet transform of the surface elevation η. The spectrum is initially relativey narrow, but widens during the evolution. It is evident that growth of sidebands is an inherent feature in the formation of the large wave events. The spectrum is characterized by substantial portions of energy being transferred to subharmonic and superharmonic wavenumbers. The energy content at the central wavenumber is reduced accordingly. Comparison between runs of different wave slope indicates that the sidebands recur with time scale proportional to $(a_0k_0)^{-2}$. We note that the theoretically most unstable sidebands appear at symmetric wavenumbers with $\Delta k/k_0 = 2a_0k_0 = 0.18$ which is obtained during the initial growth of the instabilities. This is observed in the fully nonlinear computations as well as using the NLS equation.

The role of downshifting in the large wave events: The present numerical experiment confirm what is evident from the wave tank experiments by Su (1982), namely that the formation of a downshift in long wave groups is associated with formation of envelope-solitons (within the group). While in the experiments by Su a permanent downshift was observed (including potential breaking of the waves with associated loss of mechanical energy) we find in our numerical experiments that the downshift is temporary. This downshift is responsible for the interaction taking place

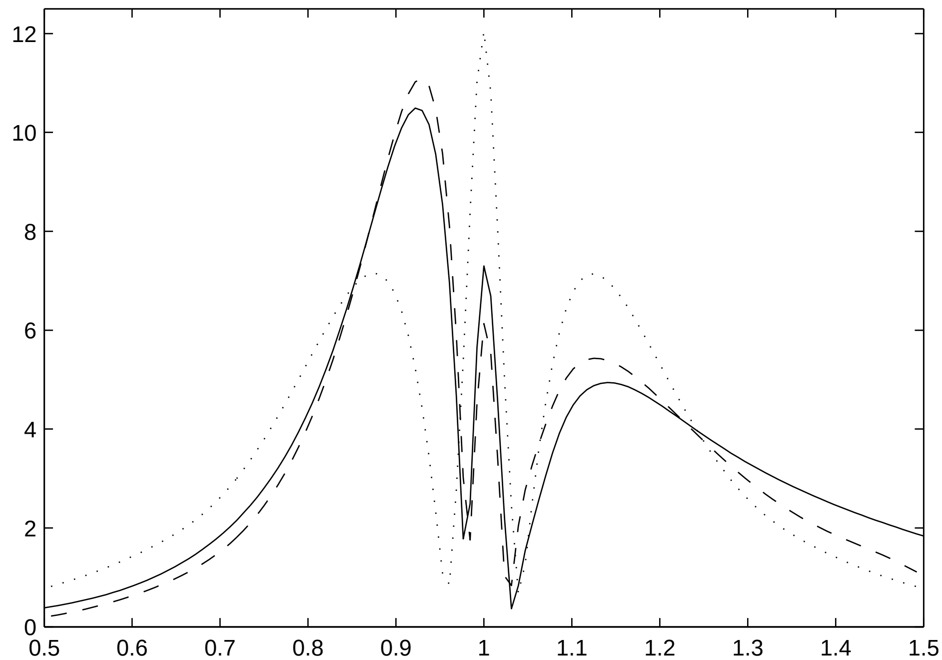

Figure 1: $|\mathcal{F}(\eta)|$ where $\mathcal{F}$ denotes Fourier transform and η the surface elevation, $t/T_0 = 155$. Fully nonlinear model (solid line), extended Dysthe equation (dashed line), NLS (dots).

between the two leading wave groups in our simulation. This wave group interaction results in the formation of the three large wave events in the simulation.

From our experiment we can generalize the result obtained by Su who found a systematic downshift for wave slopes exceeding $ak = 0.1$. We find here that the same mechanism occurs when the initial wave slope is less than 0.1 ($a_0 k_0 = 0.09$). The initial wave group is very long and shows a subsequent evolution that is highly nonlinear. We have also investigated longer wave groups with smaller slope ($a_0 k_0 = 0.046$), finding similar temporary downshifting, corresponding formation of envelope solitons and thereby formation of large wave events. Fully nonlinear simulations predict multiple interactions between envelope solitons for small wave slope. Localized and temporary downshifting of the spectrum accompanies the formation of large waves.

References

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Su, M. Y. 1982 Evolution of groups of gravity waves with moderate to high steepness. *Phys. Fluids* **25**, **12**, 2167–2174.