SPH: Towards the simulation of wave-body interactions in extreme seas

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Abstract Water entry of a solid through a free surface is of interest in ship hydrodynamics applications, namely to study ships behavior in slamming cases for instance. The present study deals with the introduction of an enhancement of SPH method, and aiming at an accurate numerical prediction of the free motion of a body in a free surface flow. The validation case exposed here is the water entry of a massive wedge, and comparisons with experimental data are provided.

INTRODUCTION

Recent advances in the field of free surface flows allowed the computation of breaking and reconnection of interface through the development of interface capturing methods such as Level Set or Volume Of Fluid. Nevertheless the inclusion of a free solid in this kind of approach remains rather complicated, due to the need of a specific treatment of the solid. "Smooth Particles Hydrodynamics" is a recent compressible Lagrangian method whose flexibility and robustness allow to solve complex free surface flows [1] [2] [3]. Concerning the computation of free solid motion, SPH avoids problems related to mesh managing but a numerical method to evaluate loads on solid boundaries had to be developed. This has been achieved through a new method consisting in calculating forces on the solid from fluid flow characteristic (Pressure, Velocity). To evaluate the accuracy of this new scheme, some results concerning a wedge water entry are provided, including comparison of dynamic condition (accelerations) with experimental data. A very good agreement is obtained, confirming the effectiveness and the accuracy of the proposed scheme. The next step consists in the simulation of steep waves interactions with a floating body. Preliminary simulations in such a situation shows a qualitatively satisfying behavior of the model, although a complete validation is still needed. As an illustration, the time evolution of a pierced box in interaction with waves is given at the end of this paper, proving the ability for this improved SPH scheme to handle some complex coupled interior-exterior fluid-solid computations such as a sinking vessel in waves.

SPH SOLVER

SPH methods are based on a set of interpolating points which are chosen in the medium. Using an interaction function (Kernel function), these points can be used to discretise partial differential equations without any underlying mesh. For free surface flows, the equations we solved are Euler equations (1, 2) and an equation of state for the pressure which is called Tait's equation (3).

$$\frac{d\boldsymbol{v}}{dt} = \boldsymbol{g} - \frac{\boldsymbol{\nabla} P}{\rho} \tag{1}$$

$$\frac{d\rho}{dt} = -\rho . \boldsymbol{\nabla} . \boldsymbol{v} \tag{2}$$

$$P = \kappa((\frac{\rho}{\rho_0})^7 - 1) \tag{3}$$

The use of this equation of state allows to avoid an expensive resolution of Poisson equation. Incompressible flows are obtained as weakly compressible flows: if the Mach number remains below 0.1 during the whole simulation, the flow can be regarded as incompressible.

The kernel function which approximates a Dirac distribution, is used to discretise previous equations through a convolution with the variables (velocity, pressure, density...).

$$W(q = \frac{|\boldsymbol{r}|}{h}) = C \begin{cases} \frac{2}{3} - q^2 + \frac{1}{2}q^3 & \text{if } 0 \leq q < 1 \\ \frac{1}{6}(2-q)^3 & \text{if } 1 \leq q < 2 \\ 0 & \text{else} \end{cases} \tag{4}$$

Particles carry all informations concerning the flow (velocity, pressure, density ...). To enhance the numerical performance, such as conservation of linear momentum, the formulae are symetrized [4] leading to the following scheme, where i-subscripted variables correspond to the i^{th} particle:

$$\frac{d\boldsymbol{x_i}}{dt} = \boldsymbol{v_i} \tag{5}$$

$$\frac{d\boldsymbol{v_i}}{dt} = \boldsymbol{g} - \Sigma_j \, m_j \, (\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}) \boldsymbol{\nabla} W(\boldsymbol{r_i} - \boldsymbol{r_j}) \tag{6}$$

$$\frac{d\rho_i}{dt} = -\Sigma_j \, m_j \, (\boldsymbol{v_i} - \boldsymbol{v_j}) \, \boldsymbol{\nabla} W(\boldsymbol{r_i} - \boldsymbol{r_j}) \tag{7}$$

In order to avoid a centered scheme which would lead to numerical instability, an artificial viscosity term Π_{ij} is added following Monaghan ([4]). This ordinary differential equation system can be integrated in time by schemes such as Runge-Kutta, Leap-Frog or Predictor-Corrector to ensure at least second order convergence in time. In this paper, a third order Runge-Kutta scheme was used.

To this point, a standard SPH scheme has been presented. This scheme has been enhanced to be able to deal with solids in free motion. It can be found in SPH related literature some applications of SPH scheme to the specific case of a wedge water entry [5].

However in this reference, the coupling between fluid and solid body motion is not solved: either the wedge is supposed to be deformable and is modeled by SPH particles with a specific equation of state which describes metal behavior, or the motion of the wedge is imposed. In the case of a deformable wedge, the computation of deformations of the body through the motion of solid particles leads to very small and restrictive time steps.

To evaluate efforts on an undeformable body, a numerical method had to be developed. This means :

1. evaluation of forces on the solid boundaries: the pressure is interpolated from the water particles which are located in the neighborhood of the solid. Indeed pressure at the boundary particles was found to be too oscillating to give satisfying results.

2. integration of the pressure effort along the solid boundaries: this is done through a low order trapezoidal rule. An increase in the order of accuracy of this quadrature formula would give better estimation of effort on the solid, this will be investigated in the near future.

3. updating of solid position and velocity: given accelerations on the solid, position and velocity are updated together with flow features using an ODE integrator (third order Runge-Kutta in this paper).

Since viscous terms are neglected in this paper, Euler equations are to be solved, and pressure loads only are evaluated.

WEDGE WATER ENTRY

These methods will be applied to the standard validation test case of a free-fall impact of a wedge. In this paper we chose the case of an asymmetric drop test of a light wedge given in [6].

At t=0 s, this free-falling wedge is dropped from 0.61 meters above the free surface with a five degrees clockwise initial heel angle and no initial velocity. Its two knuckle angles are both fitted with accelerometers dedicated to the measurement of angular and vertical accelerations. After a free fall in the air, the wedge enters the free surface. This impact generates a large deformation of the free surface and imposes a strong vertical deceleration as well as a transverse self-righting of the solid. Experiments are supposed to be realized so that no reflexion of pressure waves interacts with the wedge and the flow can be regarded as two-dimensional.

In the SPH simulation, the numerical set-up has been adapted : the tank size has been chosen to ensure no interaction between the wedge and the sound wave generated by the impact. Concerning numerical parameters, about 350,000 irregularly spaced water particles were used to achieve this computation. The smoothing length was about 14 millimeters. An adaptive time step based on

Courant condition (with a Courant number lower than 0.25), is implemented in order to reduce the computational time. The typical time step in the impact phase was of order 10^{-5} second. Since air is neglected in this SPH simulation, it was useless to simulate the whole free fall of the wedge. Thus, the initial conditions imposed in this computation are the dynamic features of the wedge at the experimental impact instant.

As can be seen in figure 1, the temporal evolution of the vertical acceleration of the wedge is well predicted in comparison with experimental data. More precisely, the maximum load at $t = 0.365$ s seems to be accurately evaluated in time as well as in amplitude in the SPH simulation. Nevertheless it should be noticed that in this SPH simulation, some features of the flow in the early stage of the impact are not well captured. Indeed, in this simulation we can see some differences at the very beginning of the impact: the slope of the temporal evolution of the vertical acceleration estimated using SPH is stiffer than in the experiment. Preliminary results of two-phase SPH simulations seem to confirm the origin of these differences to be due to air influence.

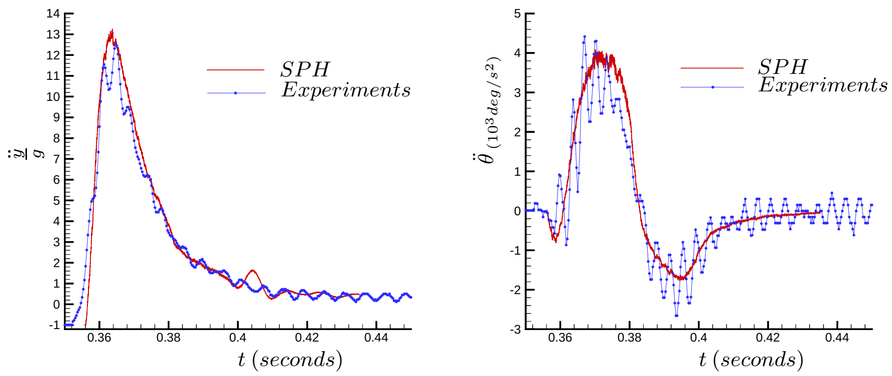

Figure 1: *Temporal evolution of the wedge vertical and angular accelerations*

The global behavior of the solid in terms of angular acceleration is also well evaluated, despite the difficulty of comparison due to the noise on the experimental signal. Note that this noise is due to structural vibrations [6], that cannot be captured by our SPH simulation since we consider the numerical wedge as non deformable.

CONCLUSION

In this paper, a new method in the field of SPH has been briefly presented. The standard SPH scheme has been enhanced with a particles sampling method which makes it able to simulate the free motion of solids in interaction with complex free surface flows including jets for instance. In the standard test case of the wedge water entry, results obtained using this approach have been compared to experimental data showing promising agreement. The main effects that occur in this specific problem are captured with a good accuracy. Future works will focus on the development of a two-phase SPH solver, in order to take into account the air-cushion effects at the very beginning of the wedge impact for instance. Furthermore, validation of this load evaluation method on other test cases would be of interest, namely once applied to industrial cases.

References

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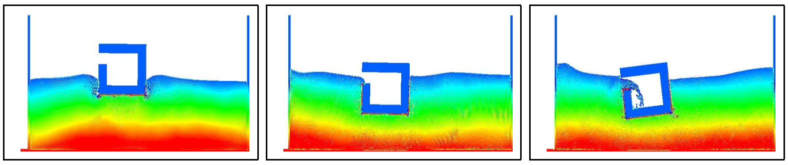
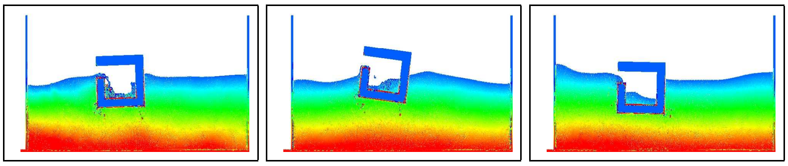
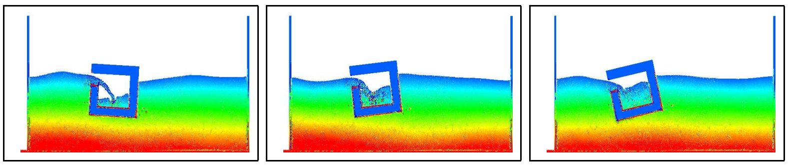


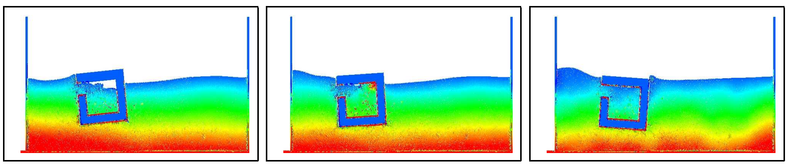
(a) $T = 0.45\ s,\ T = 0.95\ s, and\ T = 1.45\ s$



(b) $T = 1.95\ s,\ T = 2.45\ s, and\ T = 3.45\ s$



(c) $T = 3.95\ s,\ T = 4.45\ s,\ and\ T = 4.95\ s$



(d) $T = 5.45\ s,\ T = 5.70\ s,\ and\ T = 5.95\ s$

Figure 2: *Time evolution of a crude sinking vessel*