Wave crest statistics calculated using a fully nonlinear spectral response surface method

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Abstract. This paper is concerned with the calculation of the probability of exceedence of wave crest elevation. This has important implications for the design of both fixed and floating structures. In particular, the paper is interested in identifying sea-states in which so-called *freak*, or *rogue*, waves are intrinsically more likely to occur. This is investigated by ascertaining the effect that both the bandwidth and the directional spread of a spectrum has on the statistics of crest elevation. By combining a fully-nonlinear numerical wave-model with a spectral response surface method, statistics that include the full-nonlinearity of the wave-field can be determined very efficiently. It has been shown that whilst in unidirectional sea-states linear and second-order theory underestimates the probability of exceedence of crest elevation, this is not the case in most directionally spread wave-fields. Furthermore, by combining the short-term statistics relevant to sea-states with the long-term statistics of storms, the return period of the Draupner New Year's wave has been estimated.

1 Introduction

A knowledge of the statistics of wave crest elevation is fundamental for the design of most marine structures. For example, the occurrence of the largest waves determines the required air-gap and the maximum drag force on a fixed structure. Furthermore, it is important in the estimation of green-water inundation and the incidence of wave slam on floating structures. However, it has been suggested that some of the largest waves occur more often than would be predicted by linear, or second-order, theory. These waves are often referred to as *freak*, or *rogue*, waves. In this paper the short-term statistics of wave crest elevation are calculated by incorporating the fully nonlinear numerical wave-model of Bateman *et al.* (2001)

into a spectral response surface (SRS) method. The results of the short-term statistics have then been incorporated into the long-term statistical method of Tromans & Vanderschuren (1995) and the return period of a particular crest elevation at a particular location can be calculated. This method has been used to analyze hindcast data at the Draupner location and the return period of the January 1 1995 Draupner wave has been calculated. The paper begins in §2 by describing the theoretical basis and application of the SRS method. It continues in §3 by applying the method to calculate the short-term statistics of crest elevation in both unidirectional and directional sea-states. In §4 the SRS method is incorporated into that of Tromans & Vanderschuren (1995) and the return period of the Draupner wave is calculated. Concluding remarks can be found in §5.

2 SRS Method

The SRS method has been used in structural engineering to calculate the probability that a structure will fail when subjected to a number of statistically independent loads.

In this paper the SRS method is applied in order to calculate the probability of exceedence of crest elevation. A brief description of how this is undertaken follows; a more detailed description can be found in Gibson *et al.* (2005). The SRS method is applied by discretising the spectrum into a number of independent components, the statistical distribution of which is given by their variance, σ_n^2. The components are directly related to the surface elevation, and therefore, in order to consider their phasing, must be divided into η_n:

$$\eta_n = a_n \cos(\omega_n t - \varphi_n)\,, \tag{1}$$

and its Hilbert transform, $\tilde{\eta}_n$:

$$\tilde{\eta}_n = a_n \sin(\omega_n t - \varphi_n)\,, \tag{2}$$

where a_n is the amplitude of the n_{th} component, ω_n its frequency and φ_n its random phase angle. These components are then standardised, by subtracting their mean and dividing by their standard deviation, σ_n. Figure 1 shows a surface, in this case a circle, of constant probability density in the space of the standardised variables x_n and $\tilde{x}_n$. The point A represents one particular event, its response is $R(x_n, \tilde{x}_n)$, and its probability density is directly related to its distance from the origin, $\beta = \sqrt{x_n^2 + \tilde{x}_n^2}$. A first order reliability method (FORM) can then be applied in order to estimate its probability of exceedence, this is given as:

$$Q = P(\text{Crest elevation} > R(x, \tilde{x})) = \exp\left(-\frac{\beta(x, \tilde{x})^2}{2}\right)\,. \tag{3}$$

Therefore, each circle, or in more than two-dimensions each 'hyper-sphere', represents one probability of exceedence. A standard optimisation routine, such as that of Fletcher-Reeves (Press *et al.*, 1994), is then applied in order to find the maximum value of the response that corresponds to a particular probability of exceedence. In this case the response is crest elevation, which according to linear theory is given by

$$R(x_n) = \sum_{n=1}^{N} (\sigma_n x_n) \,. \tag{4}$$

However, the technique can be used for other responses, such as that of a structure subject to wave loading.

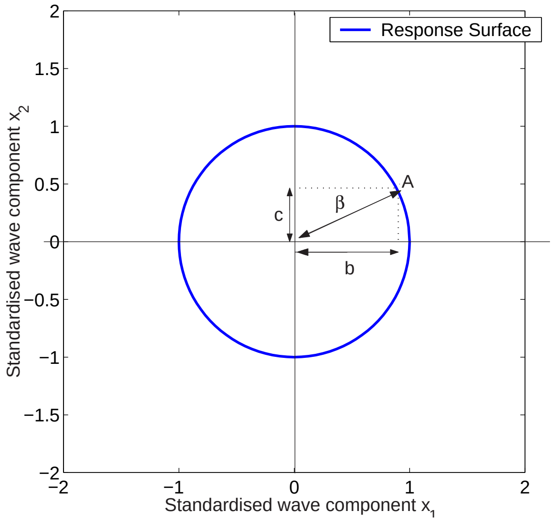

Fig. 1: A surface of constant probability density. The event A corresponds to $x_1 = b$ and $x_2 = c$ and has a probability density that is directly related to β.

3 Short-term statistics

The SRS method has been applied in order to calculate the probability of exceedence of crest elevation in both unidirectional and directional sea-states. This has been achieved using a first-order response function described in equation 4, a second-order response function based upon Sharma & Dean (1981) and described in Tromans & Vanderschuren (2002), and a fully-nonlinear response function obtained by incorporating the fully-nonlinear wave-model of Bateman *et al.* (2001) into the SRS method. The fully-nonlinear response function has been generated two ways. The first is by using the spectrum optimised linearly as the input to the wave-model and obtaining a fully-nonlinear correction to the crest elevation. This is an underestimate of the actual crest elevation as the spectrum has not been optimised fully-nonlinearly. The second method addresses this short-coming and optimises the spectrum fully-nonlinearly. Unfortunately, this is an extremely time-consuming process as the wave model, whilst efficient, must be run many times for each wave component for each probability of exceedence. Accordingly, only a few probabilities of exceedence have been optimised fully-nonlinearly in the unidirectional sea-states, and only one in the directional sea-states.

3.1 Unidirectional Seas

The SRS method has been applied to two unidirectional spectra, J1D0 and J5D0. Both are JONSWAP spectra with peak period $T_p = 12.8s$; the former has a peak enhancement factor $\gamma = 1.0$, the latter a peak enhancement factor $\gamma = 5.0$. Figure 2 shows that the fully-nonlinear results give a substantial increase in crest elevation over the linear or second-order results. This is the result of the rapid evolution of the wave spectrum discussed in more detail in Gibson & Swan (2004) and Gibson & Swan (2005). Furthermore, the figure also shows that the more narrow-banded spectrum, J5D0, is more nonlinear than J1D0, and that it is only for this spectrum that a fully-nonlinear optimisation is required. The increase in nonlinearity with reduced bandwidth is confirmed in figure 3; this indicates that the fully-nonlinear optimisation of both J1D0 and J5D0 selects a spectrum that is more narrow-banded. Further discussion and an investigation of the physical mechanisms that are responsible for these results can be found in Gibson *et al.* (2005).

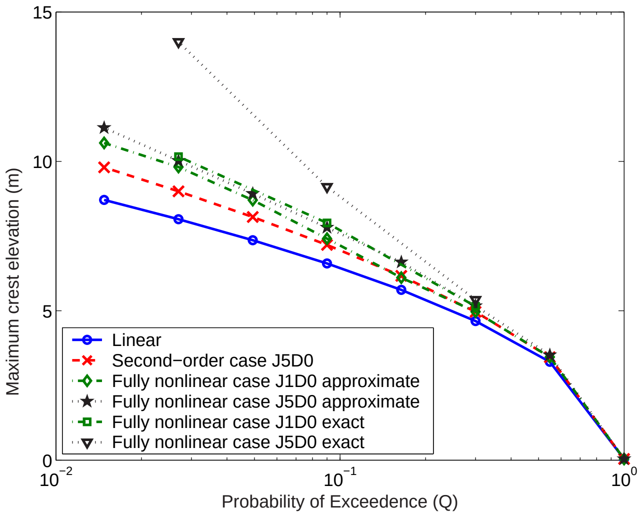

Fig. 2: The probability of exceedence of crest elevation in unidirectional seas.

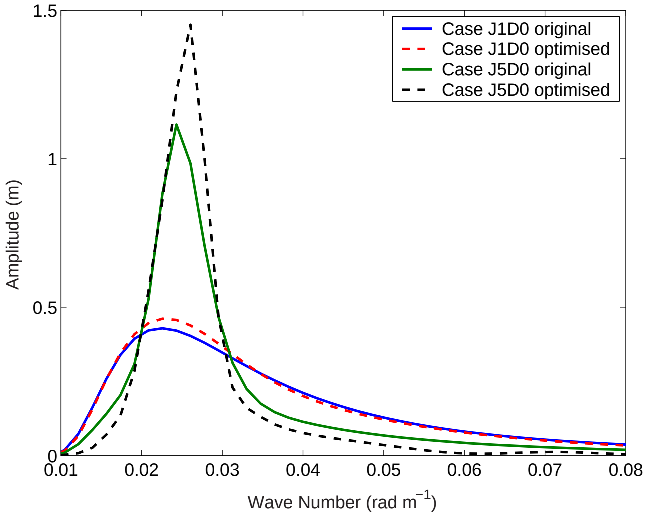

Fig. 3: The original and optimised spectra for cases G1D0 and G5D0. The spectra are coarsely discretised due to the time consuming nature of the optimisation process.

3.2 Multi-directional Seas

The SRS method has also been applied to two directional spectra, J5D10 and J5D30. Once again both are JONSWAP spectra with peak period $T_p = 12.8s$ and peak enhancement factor $\gamma = 5.0$; in the former the standard deviation of the wrapped normal directional spreading function is $\sigma_\theta = 10^\circ$, in the latter it is $\sigma_\theta = 30^\circ$. Figure 4 indicates that for both spectra the fully-nonlinear crest elevation is less than the second-order prediction. Furthermore, the fully-nonlinear optimisation of the wave spectra only marginally increases the crest elevation. This suggests that the focussing of wave components is not a mechanism by which *rogue* waves can form in sea-states characterised by a directionally spread JONSWAP spectrum. This is the result of the balance between the spectral evolution, involving changes in both the frequency and the directional spectrum, and the focussing of the wave components discussed in more detail in Gibson & Swan (2004) and Gibson & Swan (2005).

Figure 5 shows the probability of exceedence of crest elevation for a directionally spread Gaussian spectrum of $T_p = 16s$ and $\sigma = 5^\circ$, characteristic of swell-dominated sea-states. In this case the crest elevations are very much larger than those of the JONSWAP spectrum. However, figure 5 also shows that in mixed sea-states, those in which there is both a wind (JONSWAP) and a swell (Gaussian) component, the result is similar to that for a purely wind-dominated (JONSWAP) sea.

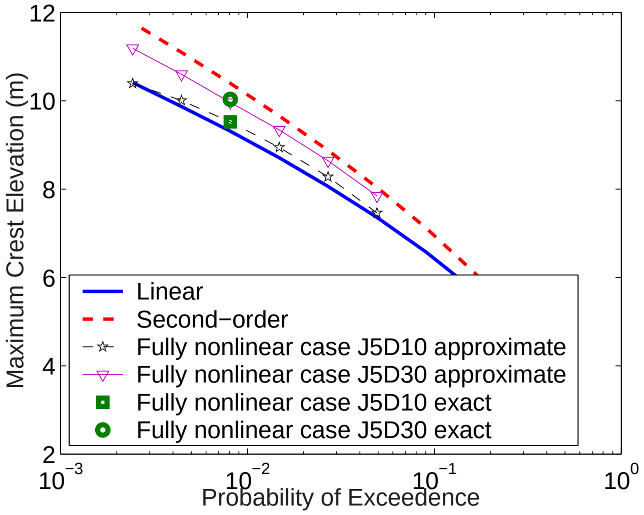

Fig. 4: The probability of exceedence of crest elevation in directional seas.

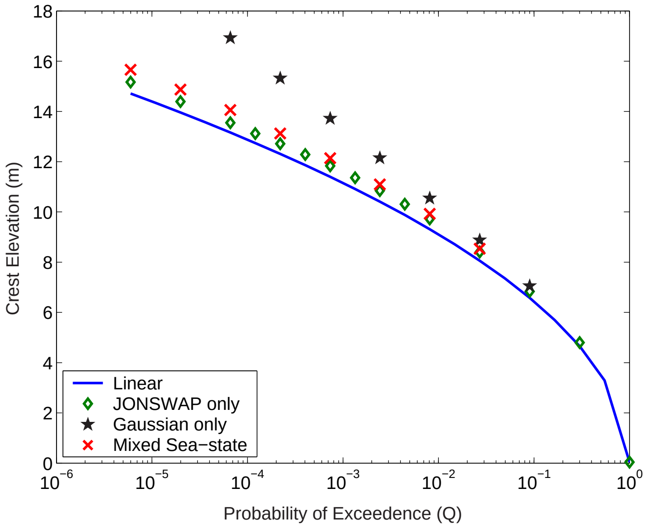

Fig. 5: The probability of exceedence of crest elevation in wind dominated, swell dominated and mixed directional-spread sea-states. In the case of the mixed sea the amplitude has been split evenly between the wind and swell components.

4 Long-term statistics

The results of the previous section relate to the short-term statistics of sea-states. However, the design of a marine structure typically requires knowledge of the long-term statistics at a particular location. This can be undertaken by applying Tromans & Vanderschuren (1995). The method considers storms as independent events and requires two types of distribution to be defined. The first is the long-term statistics of storms, which is calculated from measured or hindcast data. The second is the short-term statistics of waves within a sea-state, which can be calculated using the SRS method. Tromans & Vanderschuren (1995) has been applied, in conjunction with the SRS method and using hindcast data for the Draupner location, in order to calculate the return period of an 18.5m crest elevation; corresponding to the New Year Wave recorded on January 1 1995. The manner in which this has been undertaken is described below.

If the spectrum is assumed to be JONSWAP then it is possible to construct a transfer function from linear to nonlinear crest elevation:

$$C_{nl} = C_{lin}.F(C_{lin}K_p)\,, \tag{5}$$

where C_{lin} is the linear crest elevation, with a probability of exceedence defined by the Rayleigh distribution, C_{nl} is the nonlinear crest elevation and K_p is the wave-number of the peak of the spectrum. This ignores the parameters of the spectrum as previous results (Gibson & Swan, 2005) have indicated that, in directional seas, they have little effect on the distribution of crest elevation. The form of the transfer function, for a JONSWAP spectrum, is as follows:

$$F(C_{lin}K_p) = 1 + \beta(C_{lin}K_p)^p + \gamma(C_{lin}K_p)^q\,, \tag{6}$$

where p and q are approximately one and two respectively. Using these values, the β term corresponds to the second-order bound correction, whilst the γ term corresponds to the third-order resonant change to the amplitude of the underlying linear spectrum. The various coefficients are given in table 1 and the fit to the data described in figure 6.

Parameter	**Second-Order**	**Fully-Nonlinear**
β	0.50	0.87
γ	0.0	-1.63
p	1	1.17
q	2	1.92

Table 1: Parameters of the transfer functions from linear to nonlinear crest elevation using Equations 6

Tromans & Vanderschuren (1995) can now be applied using linear theory with a nonlinear transfer function utilised at each step. Figure 7 depicts the return period of crest elevation at the Draupner location calculated using linear, second-order and fully-nonlinear theory. This shows that the best estimate for the return period of the New Year wave recorded at the Draupner platform wave is 800 years; which is greater than the 300 years calculated using a second-order model.

5 Concluding Remarks

In this paper both short- and long-term statistics of crest elevation have been considered. This has been undertaken by incorporating the fully-nonlinear wave-model of Bateman *et al.* (2001) into both the SRS method and that of Tromans & Vanderschuren (1995). The results have shown that whilst in unidirectional sea-states the fully-nonlinear crest elevations are significantly larger than those calculated to second-order, this is not necessarily the case in directional seas. Indeed, in broad-banded directionally spread seas, characterised by a JONSWAP spectrum, the fully-nonlinear crest elevations are in fact lower than those predicted to second-order. However, in narrow-banded swell-dominated seas, characterised by a Gaussian spectrum with a small spreading parameter, the results are similar to those of the unidirectional simulations; with crest elevations significantly higher than second-order theory. This suggests that it is in swell-dominated sea-states that *rogue* waves are intrinsically more likely to occur.

The SRS method has been used in conjunction with that of Tromans & Vanderschuren (1995) in order to calculate the return period of the Draupner wave. This has been undertaken by employing a transfer function from the linear to the fully-nonlinear results. The result gives a return period of a 18.5m crest elevation of 800 years.

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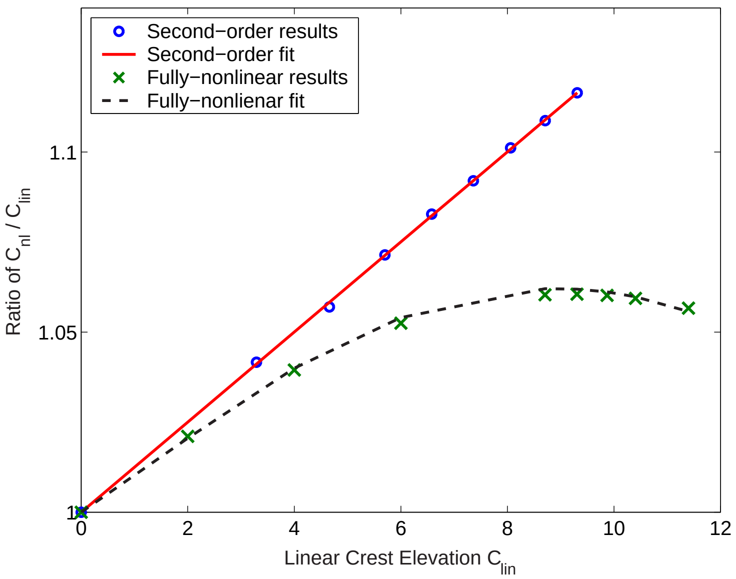

Fig. 6: The fit to the nonlinear crest elevations. In terms of C_{lin}, the fitting function is parabolic for second-order results and cubic for the fully nonlinear results; the latter indicating the dominance of third-order terms.

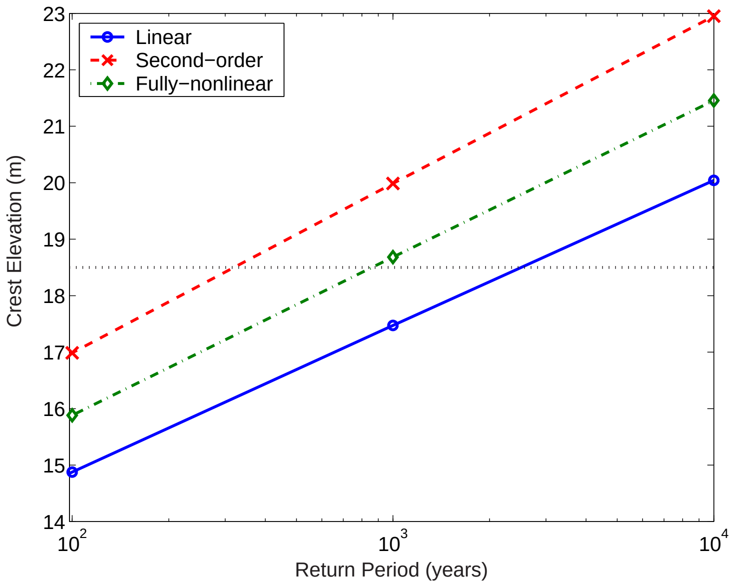

Fig. 7: The return period of crest elevation at the Draupner location.

Bibliography

Bateman, W. J. D., Swan, C. & Taylor, P. H. 2001 On the efficient numerical simulation of directionally-spread surface water waves. *Journal of Computational Physics* **174**, 277–305.

Gibson, R. S. & Swan, C. 2004 The role of resonant wave interactions in the evolution of extreme wave events. *Rogue Waves 2004* .

Gibson, R. S. & Swan, C. 2005 The evolution of large ocean waves: the role of local and rapid spectral changes. *Submitted to the Royal Society* .

Gibson, R. S., Swan, C. & Tromans, P. 2005 Fully-nonlinear statistics of wave crest elevation calculated using a spectral response surface method: Applications to unidirectional sea-states. *Submitted to the Journal of Physcial Oceanography* .

Press, W. H., Flannery, B. P., Teukolsky, S. A. & Vetterling, W. T. 1994 *Numerical Recipes in C—The Art of Scientific Computing.*. Cambridge University Press.

Sharma, J. N. & Dean, R. G. 1981 Second-order directional seas and associated wave forces. *Society of Petroleum Engineering Journal* **4**, 129–140.

Tromans, P. S. & Vanderschuren, L. 1995 Response based design conditions in the North Sea: application of a new method. In *OTC 7683, Offshore Technology Conference, Houston*.

Tromans, P. S. & Vanderschuren, L. 2002 The statistics of wave crest elevation calculated to second order. In *Proceedings of OMAE02 Conference*.