SELF-SIMILARITY OF WIND-WAVE SPECTRA.

NUMERICAL AND THEORETICAL STUDIES

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This is the only slide where terms *“Rogue Waves”*, *“Freak Waves”* etc. appear

To study extreme, abnormal waves one has to know what `a normal' state is

- Self-similarity is implied in experimental parameterizations of wind-wave spectra;
- Self-similarity is an inherent feature of `normal' wind-wave field

JONSWAP spectrum

$$E(\omega) = \frac{\alpha g^2}{\omega_p^5} \left(\frac{\omega_p}{\omega}\right)^4 \exp\left(-\frac{\omega_p^4}{\omega^4}\right) \gamma^{\exp\left(\frac{(\omega-\omega_p)^2}{2\sigma_p^2\omega_p^2}\right)}$$

ω / ω_p «internal» parameter to describe spectral form

U_h / C_p «external» - wave age - effect of external forcing

$$\alpha = \alpha_0 \left(U_h \omega_p / g \right)^{\kappa_\beta}$$

No explicit dependence on duration (t) or fetch (x)

$$E_{tot} \sim t^p \ (x^p); \quad \omega_{mean} \sim t^{-q} \ (x^{-q})$$

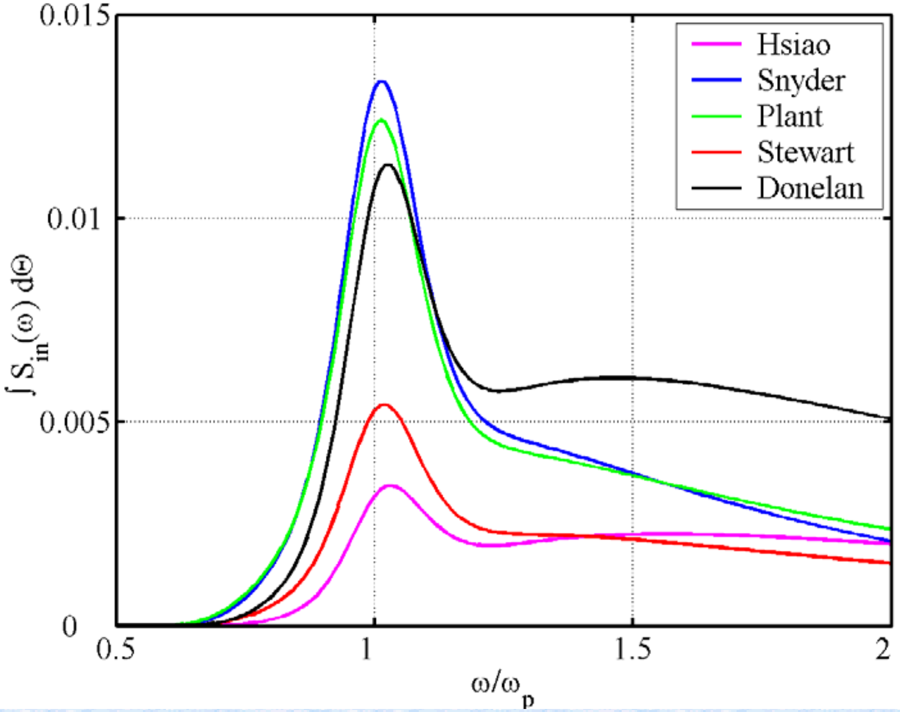
The Hasselmann equation

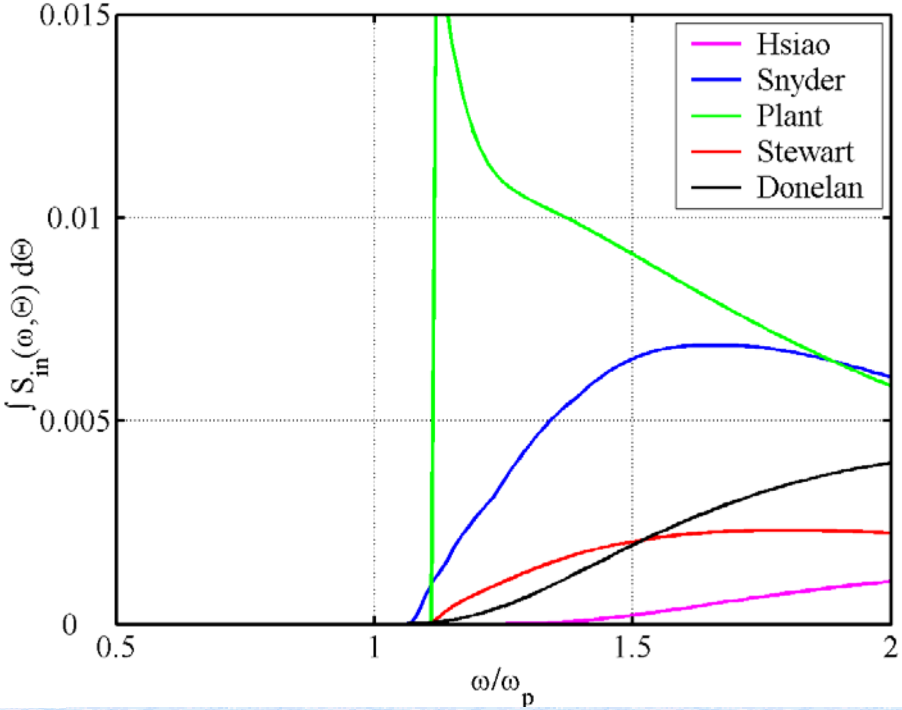
$$\frac{dn_k}{dt} = S_{nl} + S_{input} + S_{diss}$$

$$S_{nl} = 2\pi \int |T_{0123}|^2 (n_0 n_2 n_3 + n_1 n_2 n_3 - n_0 n_1 n_2 - n_0 n_1 n_3)$$
$$\times \delta(\omega_0 + \omega_1 - \omega_2 - \omega_3)\delta(\mathbf{k}_0 + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3$$

S_{input}, S_{diss} - empirical parameterizations

What parameterization of wave input is true?





"Young" waves $U/C_p=2$

"Old" waves $U/C_p=0.9$

Our key point

Nonlinearity dominates !

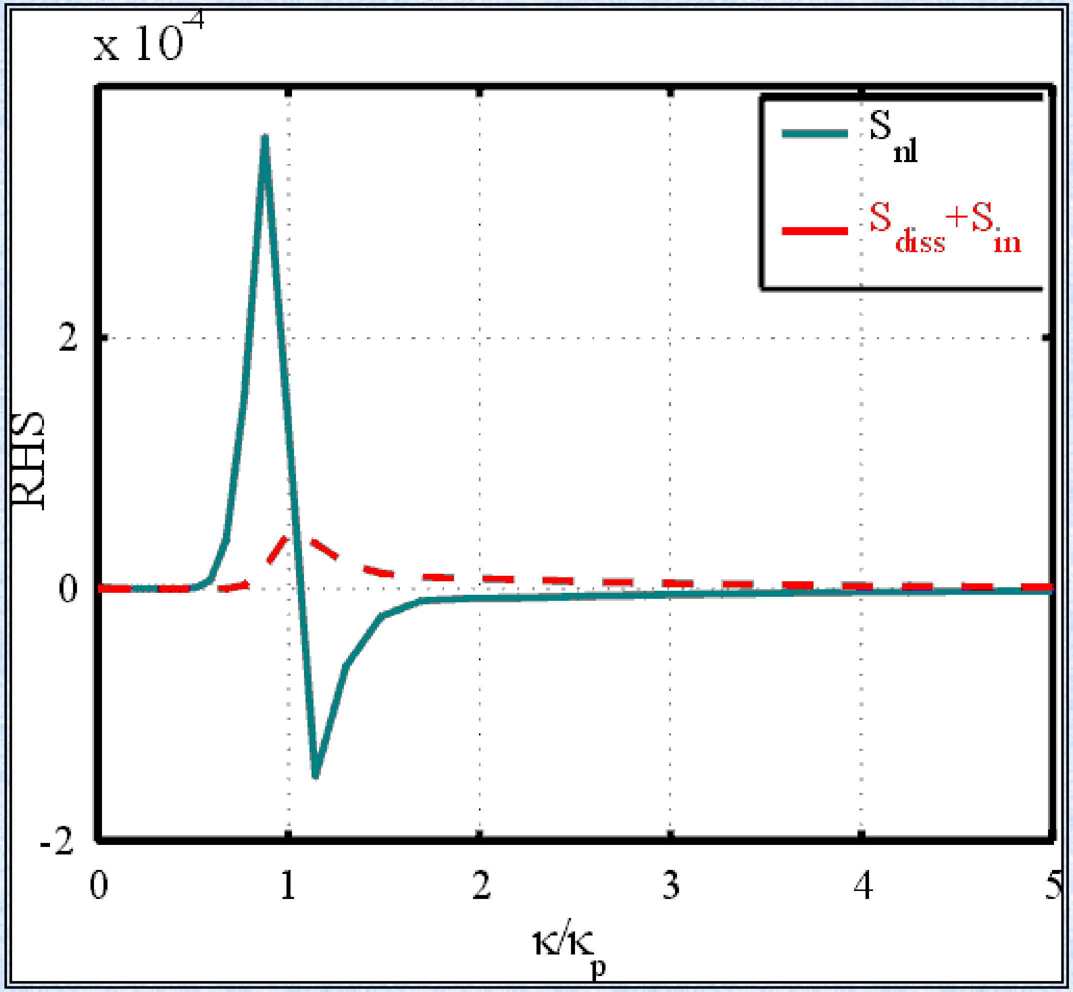
$$S_{nl} >> S_{input}\,,\ \ S_{diss}$$

There is no characteristic scale (deep water waves), i.e. homogeneity of the collision integral gives

$$S_{nl} \sim \mathbf{n^3\, k^{19/4}}$$

"Primitive" comparison of terms in KE

S_{nl} *vs* $S_{diss}+S_{in}$; t=1.5 hours



Look for approximate solutions in a self-similar form

(quite similarly for fetch-limited case)

$$n = at^{\alpha} U_{\beta}(b\mathbf{k}t^{\beta}, t); \quad \boldsymbol{\xi} = b\mathbf{k}t^{\beta}; \quad a = b^{19/4}; \quad \alpha = \frac{19\beta - 2}{4}$$

After substitution

$$\frac{\partial U}{\partial t} \quad 0 = S_{nl}\left[U_{\beta}(\xi)\right] - \beta\,\xi \nabla_{\xi} U_{\beta} - \alpha U_{\beta} + \frac{S_{in} + S_{diss}}{at^{\alpha - 1}}$$

if $\alpha > 1$ the effect of S_{in} and S_{diss} vanishes

at $t \rightarrow \infty$

Determine parameters α, β from the higher-order approximation

$$N \sim \int t^{\alpha} U(\mathbf{k} t^{\beta}) d\mathbf{k} \sim t^{r}$$

$$r = \alpha - 2\beta$$

r is an exponent of wave action growth and is the only parameter of the family of S-S solutions

$r = 0$ – swell (no external input)

$r = 1$ – constant wave action input

$r = 4/3$ – constant wave energy input

Algebra <-> Physics

We split wind-wave balance into two parts

$$\frac{dn_k}{dt} = S_{nl}$$

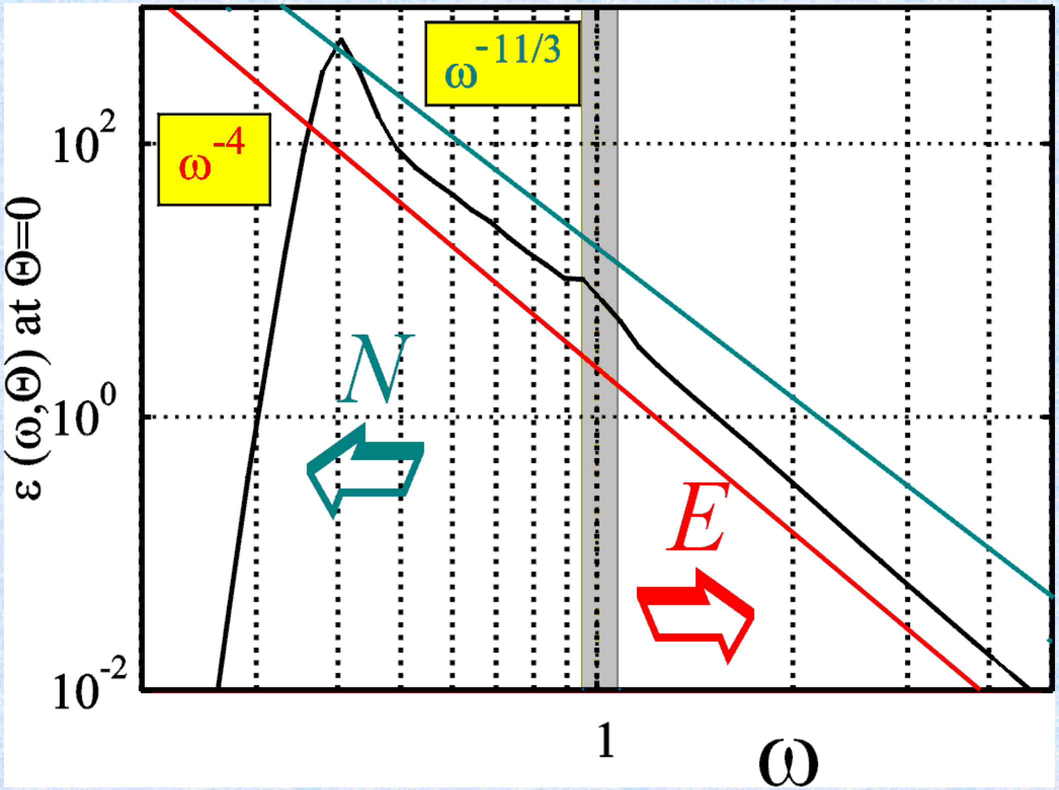
A form of the self-similar solution

$$\frac{d\langle n_k \rangle}{dt} = \langle S_{in} \rangle + \langle S_{diss} \rangle$$

Integral balance for the wind-driven waves

Does this model work?

Kolmogorov's cascades
(Zakharov, PhD thesis 1966)



$$E^{(1)}(\omega,\theta) = C_p \frac{g^{4/3} P^{1/3}}{\omega^4}$$

Direct cascade (Zakharov & Filonenko 1967)

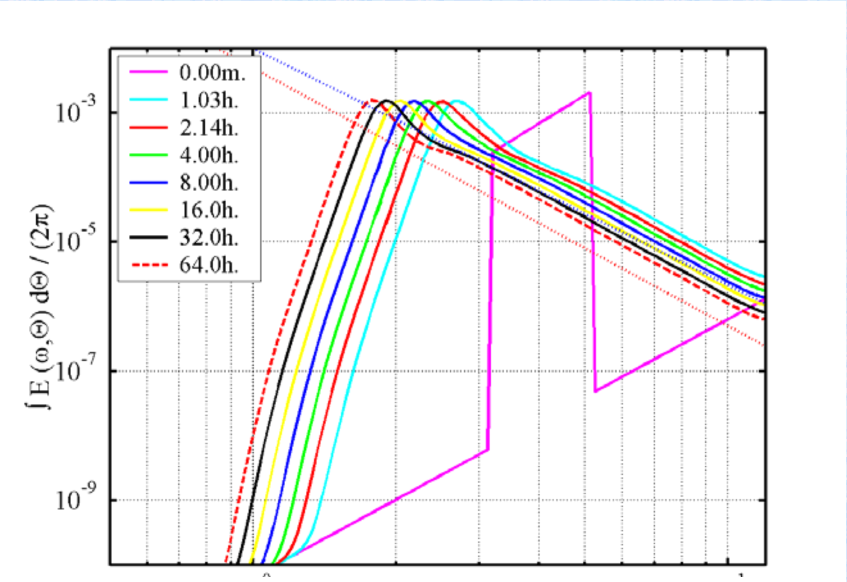
$$E^{(2)}(\omega,\theta) = C_q \frac{g^{4/3} Q^{1/3}}{\omega^{11/3}}$$

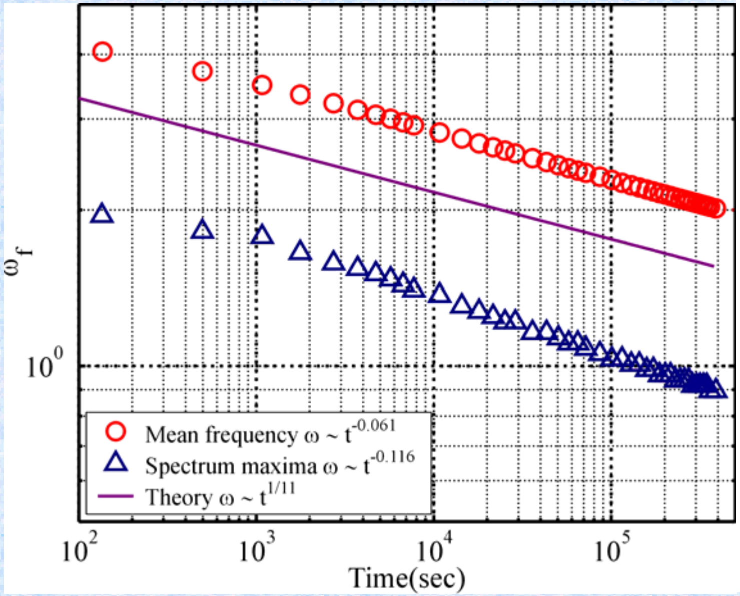
Inverse cascade (Zakharov & Zaslavskii 1983)

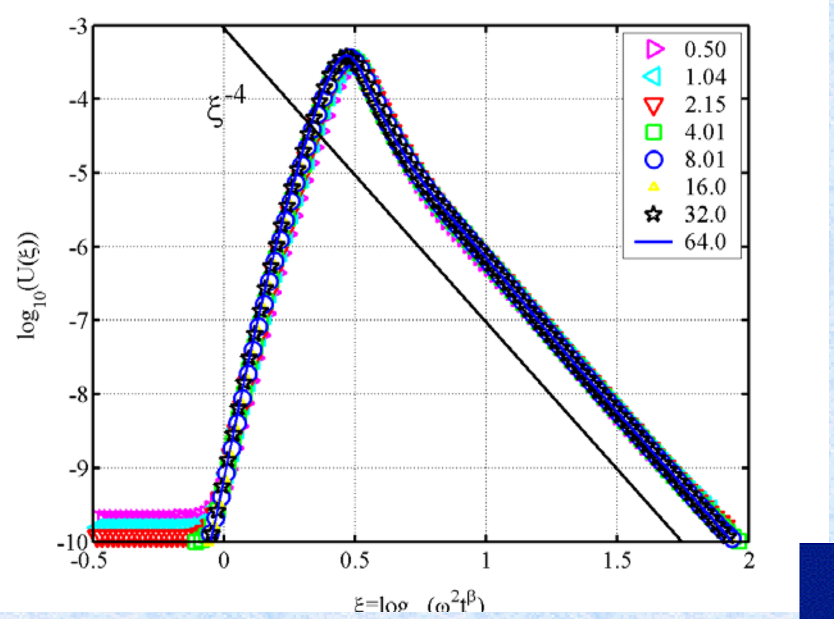
Does numerical solutions satisfy self-similar scaling?

Swell

$$n = t^{2/11} U_0(\omega^2 t^{1/11}, \Theta)$$

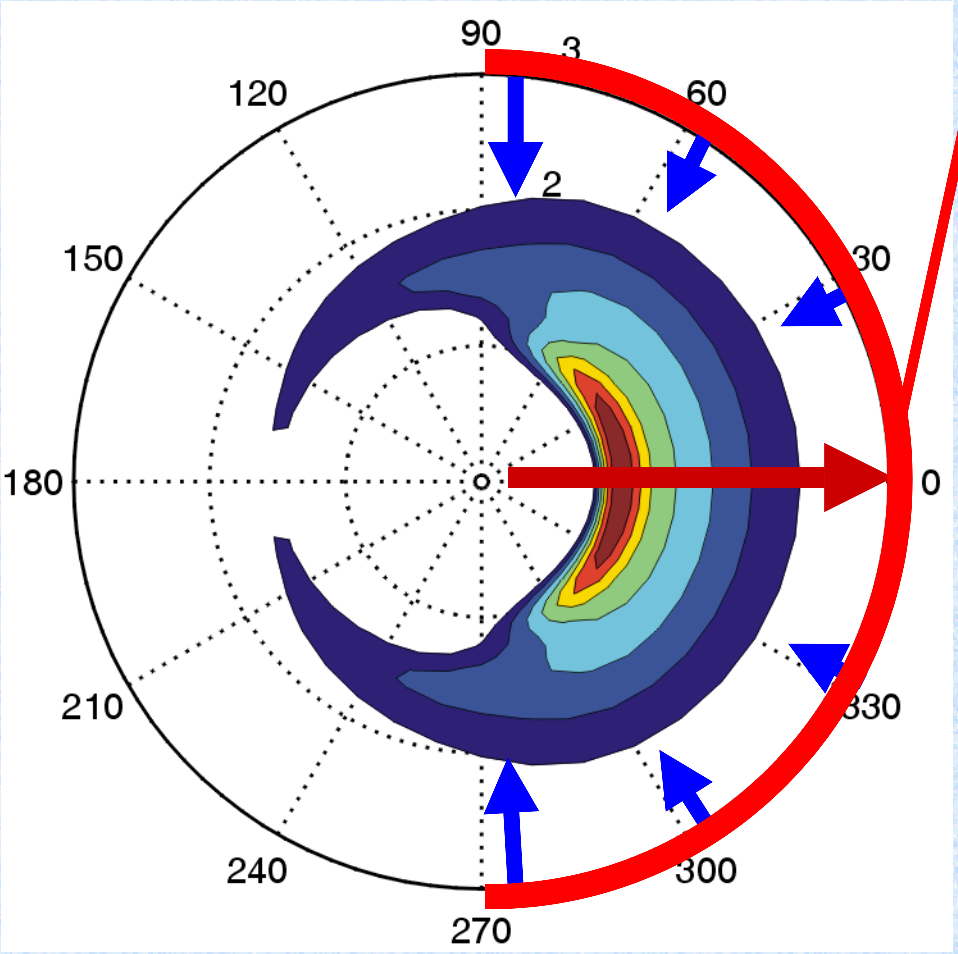






“Academic” pumping $N_{tot} \sim t^r$

(just to obtain self-similarity in the ‘pure’ state)



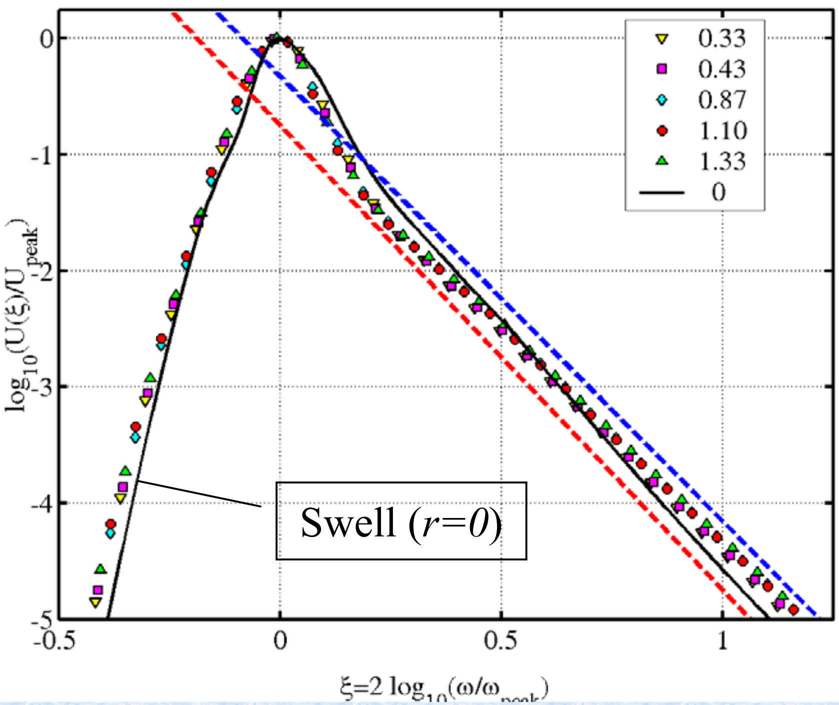
Wind-wave increments depend on time !!!

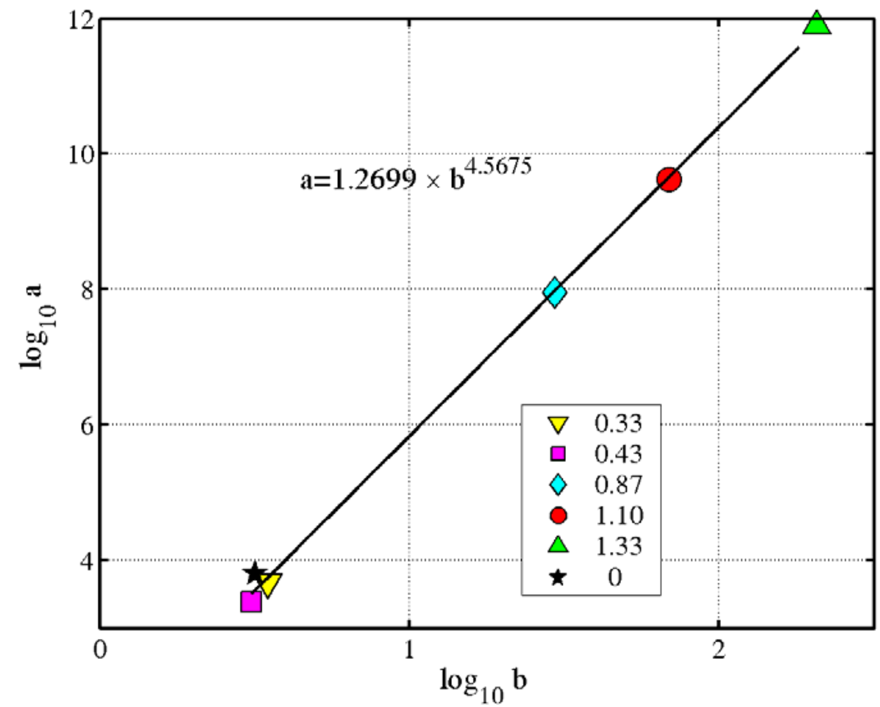
For sea waves $r \approx 1$

In our runs $1/3 < r < 4/3$

Compare "form" functions $U(\xi)$ for different growth exponents r (`academic')

$$(\, n = a t^{\alpha} U_r(b\omega^2 t^{\beta})\,,\theta\,)$$

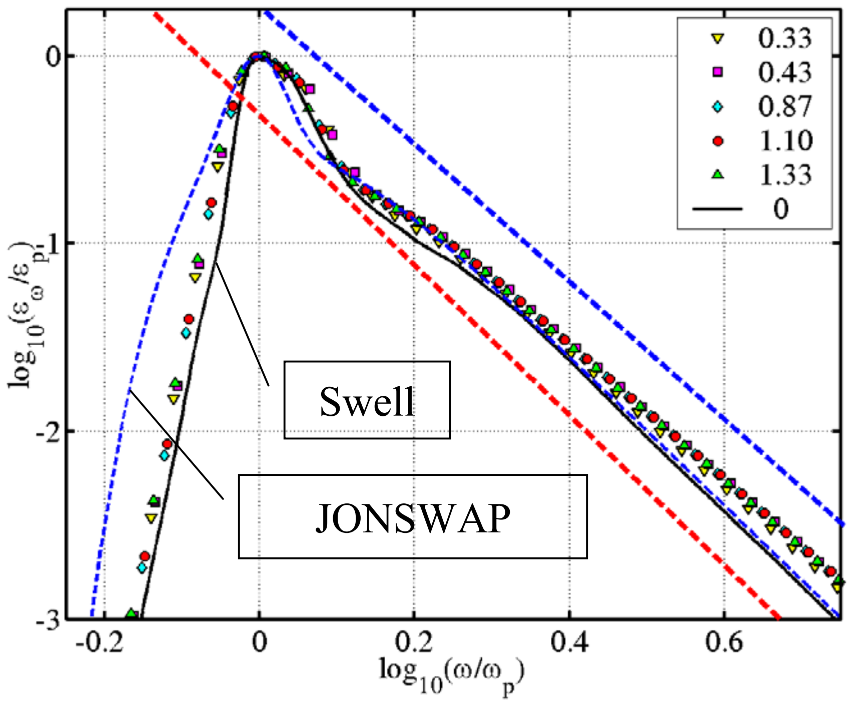


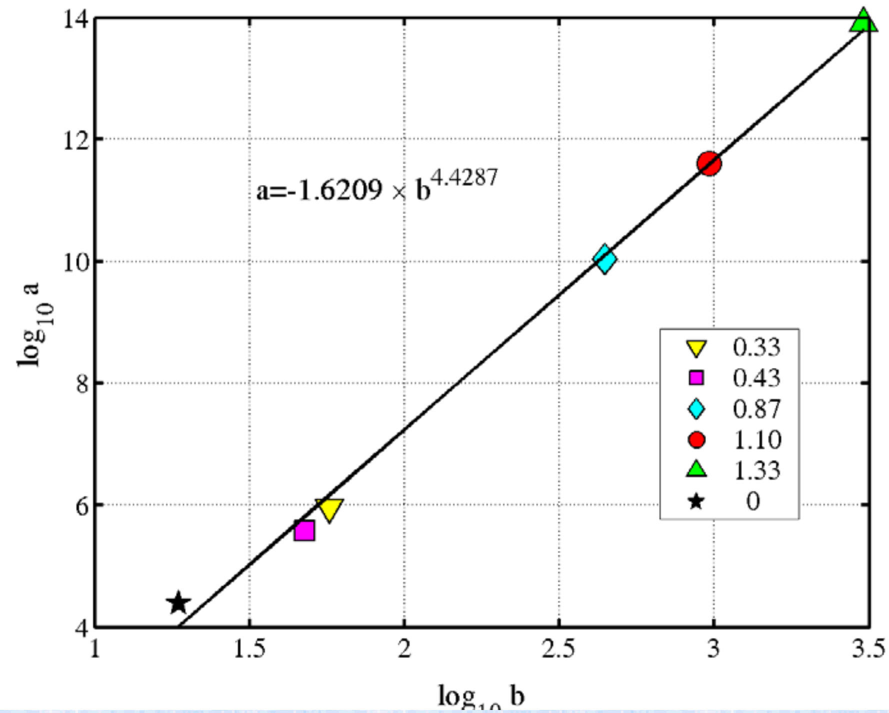


- Numerical solutions keep self-similar scaling $a \sim b^{19/4}$
- "Form" functions $U(\xi)$ does not depend on r?

Scaling of frequency spectra (direction-averaged) for different wave growth rates r

$$(n_k = a t^{\alpha} U_r(b\mathbf{k}t^{b}))$$

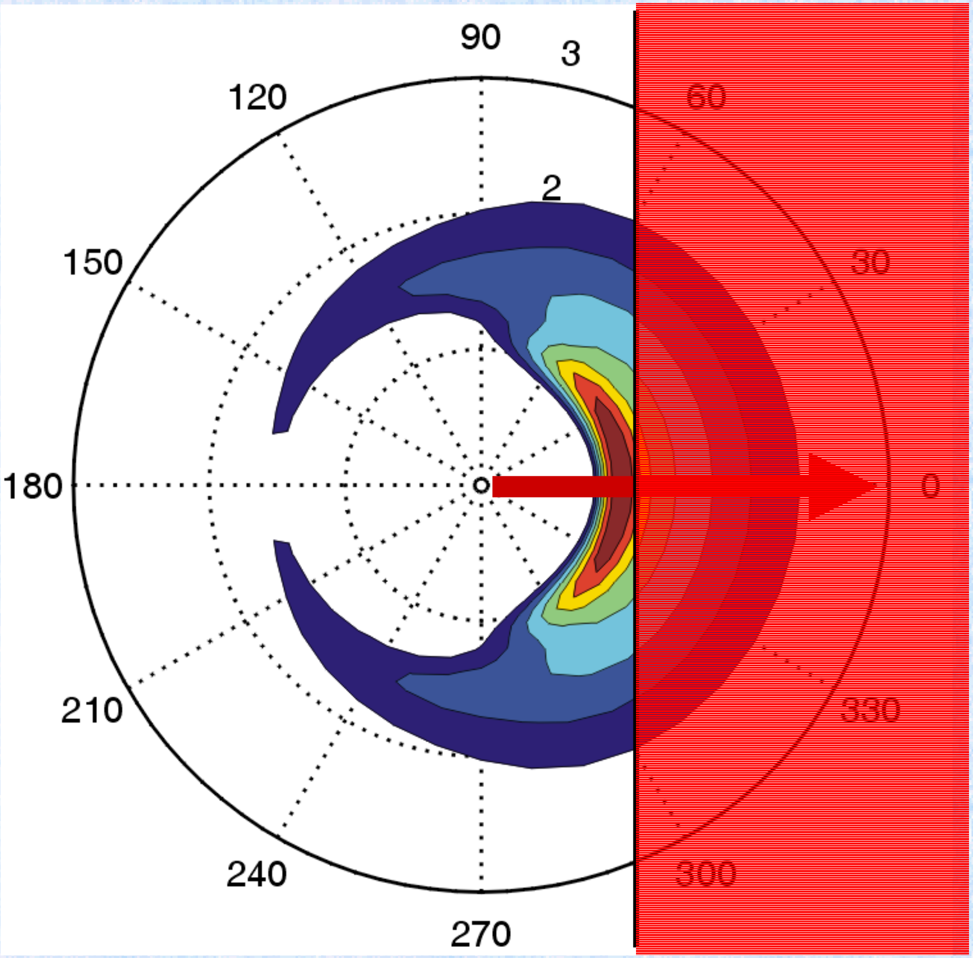




- Frequency spectra keep self-similar scaling $a \sim b^{19/4}$
- $E(\omega)/E_p$ does not depend on r ?

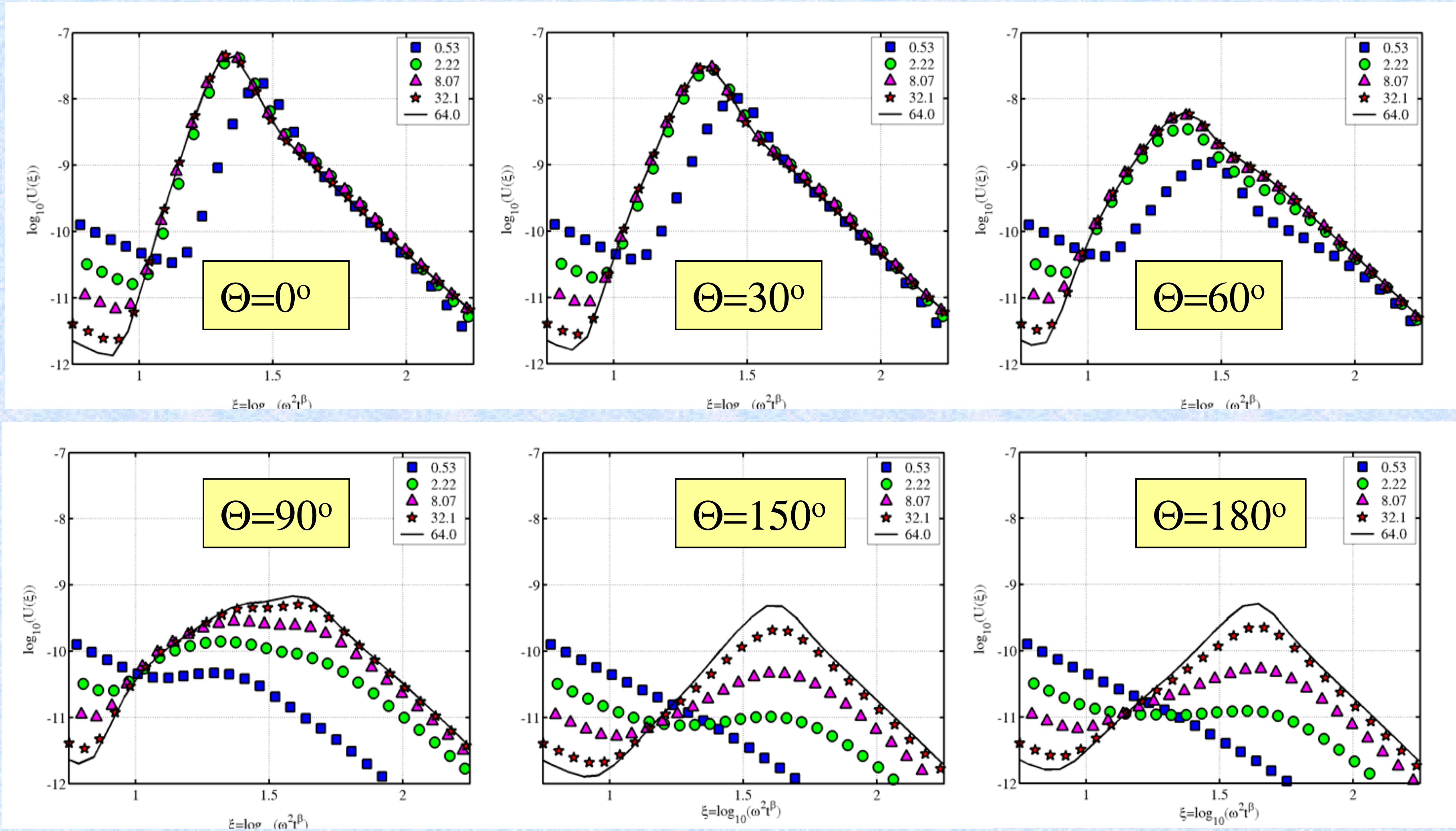
Waves under wave pumping:

No room for inertial interval – no room for self-similarity?

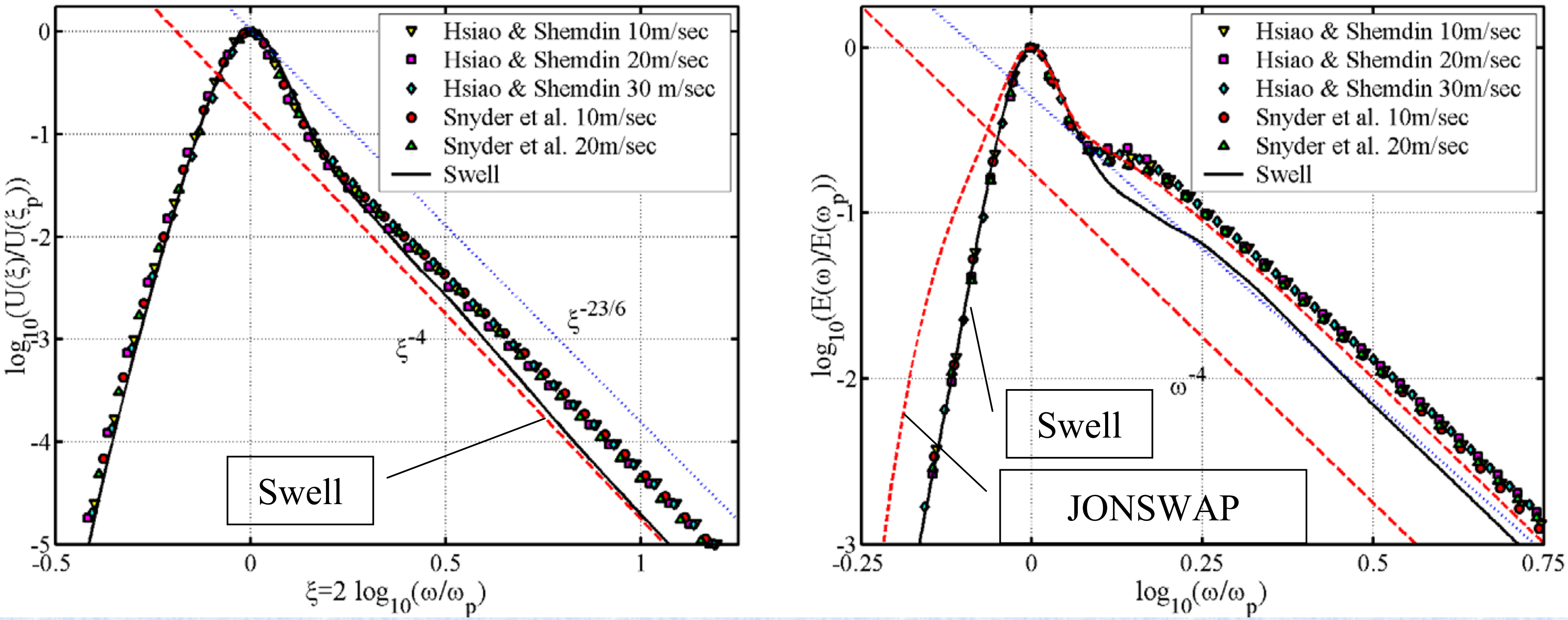


“Form” functions $U(\xi)$ for waves under wind pumping

$$\xi=\omega^2 t^\beta, \quad n=t^\alpha U(\xi,\Theta)$$



Down-wind (left) and frequency (right) spectra for `real' wave pumping



Experimental spectra have self-similar forms !!!

- JONSWAP spectrum

$$\frac{E(\omega/\omega_p,\Theta)\,\omega_p^5}{g^2} = \frac{\alpha_0}{2\pi}\left(\frac{U_{10}}{C_p}\right)^{\kappa_\alpha} U_0\left(R\frac{\omega^2}{\omega_p^2},\Theta\right)$$

- Self-similar solutions can be expressed in terms of mean (**peak?**) frequency and mean (**peak?**) energy quite similar to the experimental forms

$$\frac{E(\omega/\omega_p,\Theta)\,\omega_p^5}{g^2} = a(\omega_p/gU)^{\kappa_\alpha}\,U_r(b\frac{\omega^2}{\omega_p^2},\Theta)$$

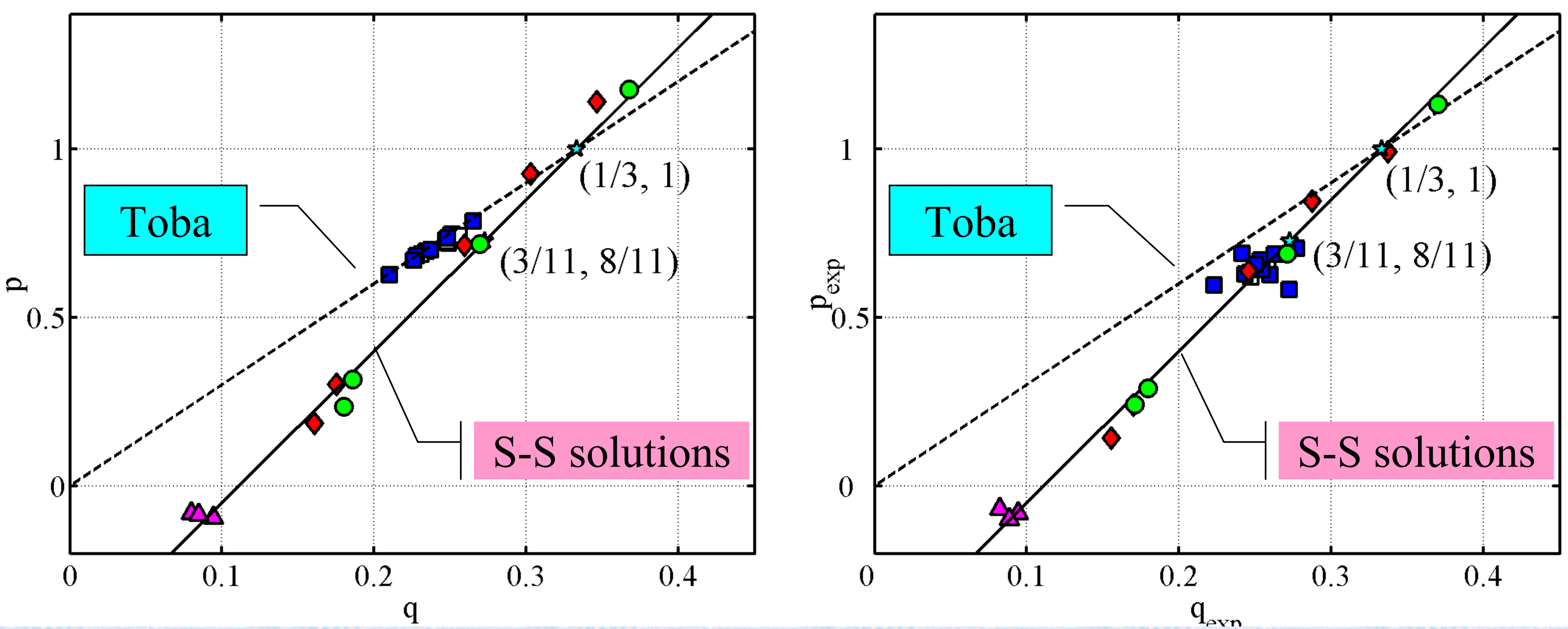
Exponents of wind-wave growth

How to compare correctly numerical solutions and experimental parameterizations?

There is a self-similar `core' and non-self-similar background

- $E_{tot} \sim t^{p}$ **or** $E_{peak} \sim t^{p}$
- $\omega_{mean} \sim t^{-q}$ $E_{peak} \sim t^{p}$

Exponents of energy growth *p* and frequency downshift *q* for **mean (left)** and **peak (right)** values.



`Academic' series

Anisotropic generation

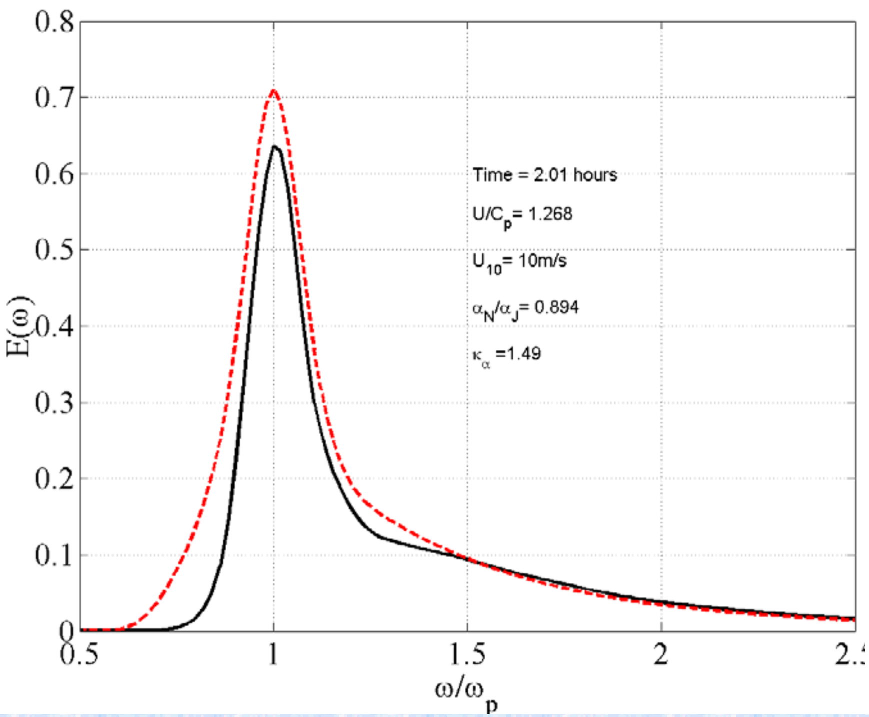
Isotropic generation

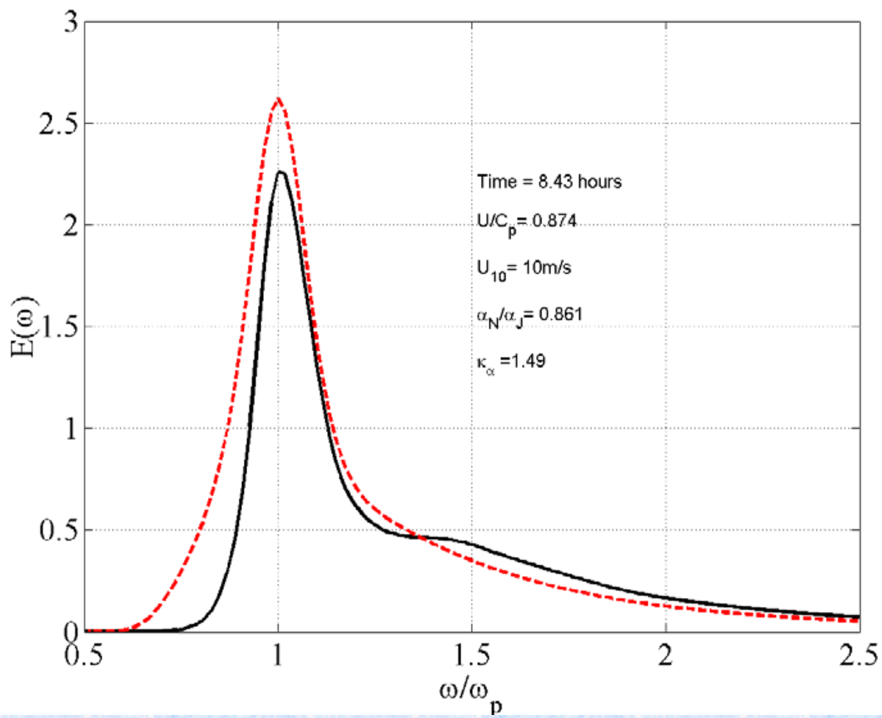
`Realistic' inputs

Wave input

Swell

Numerical frequency spectra for young (left, U/C_p=1.3) and old waves (right, U/Cp=0.87). JONSWAP spectra (dashed) use standard set of parameters. Characteristics of peak growth are given for the comparison. Wave input Donelan & Pierson-jr. (1987)





Summary

- There is a strong tendency of wind-wave spectra to self-similar behaviour in a wide range of wind-wave conditions;
- The forms of the spectra are close to universal that is consistent with experimental parameterizations of wind-wave spectra
- Self-similar `core’ of wind-wave spectra coexists with non-self-similar background. Peak parameters are more adequate to spectra description