We present a novel variational Wave Acquisition Stereo System (WASS) that exploits new stereo reconstruction techniques for accurate estimates of the spatio-temporal dynamics of extreme oceanic waves. We shall present a statistical analysis that explains the role of nonlinearities in the formation of large waves. When nonlinearities are negligible, ocean waves are usually modeled by the linear superimposition of a large number of elementary waves having amplitudes related to a given spectrum and random phases. In this case, large waves are due to the linear focussing of a wave group (Lindgren 1972, Boccotti 2000) and their crests and troughs are both Rayleigh distributed.

If second-order nonlinearities are dominant, then the sea surface displays sharper narrower crests and shallower more rounded troughs. As a result, the skewness of surface elevations is positive, and wave crests are distributed according to the Tayfun model (Tayfun 1980). If, however, elementary waves also exchange energy nonlinearly via third order four-wave resonances, narrowband wave trains can undergo intense modulational instability enhancing the occurrence of large waves (Janssen 2003, Fedele 2008) and the distribution of crest heights deviates from the Tayfun model. This is confirmed by the wave-flume experiments in (Onorato et al. 2006) and the numerical simulations of the Dysthe equation (Socquet-Juglard et al. 2005), a special case of the Zakharov equation (Zakharov 1999). However, in broadband waves the Tayfun distribution appears to explain crest statistics well (Fedele 2008, Tayfun & Fedele 2007). The unusually wave crests observed in both the latter experiments and simulations are well explained by a recent Gram-Charlier approximation of the crest distribution proposed in Tayfun & Fedele 2007, which is based on heuristic arguments.

The derivation of this crest model stems from the general Hermite expansion of random variables, and it does relate to the physics of nonlinear waves only through the statistical estimations of both the skewness and kurtosis of the wave surface. Could such type of Gram-Charlier crest models be derived directly from the basic equations governing the ocean dynamics, without any use of Hermite-type expansions?

An answer to this question is provided. We present the formulation of a new model of stochastic wave groups which provides a theoretical framework for the non-Gaussian statistics of large waves in oceanic turbulence (Zakharov 1999). Stochastic wave groups describe the dynamics of the wave surface around a randomly chosen large crest (Lindgren 1972, Boccotti 2000), and their nonlinear space-time evolution reveals the statistical structure of large wave crests and thus their expected shape. A generalization of the Tayfun model for the statistical distribution of crest heights over large waves is then derived. Experimental video data obtained via the variational WASS (Gallego et al. 2008) are in agreement with the new model.

REFERENCES

Figure 1: Input stereo pair images to the algorithm. The rectangular domain (8 m x 8.7 m) of the reconstructed surface or elevation map (right column) has been superimposed. The height of the waves is in the range ±0.2 cm.

Figure 2: (left) Reconstructed wave surface from VWASS normalized with zero mean and unit variance; (right) normalized wave spectrum.

Figure 3. (left) Probability density p(\(\eta\)) of the reconstructed wave surface \(\eta\); (right) Wave spectrum \(S(k)\) as function of the wave number \(k\) computed from the reconstructed wave surface \(\eta\) in Figure 4. The spectrum tail decays as \(k^{-2.5}\) in agreement with wave turbulence theory (Zakharov 1999, Socquet-Juglard et al. 2005).