First order Lagrange models for asymmetric ocean waves

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Abstract. The first order stochastic Lagrange wave model is a simple and realistic alternative to the linear Gauss wave model. However, while the produced waves are crest-trough asymmetric, they are still statistically front-back symmetric. This symmetry depends on a simple correlation structure between the vertical and horizontal components in the simplest Lagrange model. We present a physically motivated modification that is able to generate also front-back asymmetry. We further give expressions for the exact statistical distributions of several important slope characteristics that illustrate the asymmetry, for instance the space slope at the time of a level crossing.

Introduction

Wave front steepness is one of the many characteristics that have to be considered in the safety analysis of marine vessels and structures. In general, measures of crest-trough and front-back asymmetry of irregular waves are notoriously difficult to derive from the common stochastic wave models. Empirically, asymmetry properties has drawn considerable interest, in particular when more or less continuously sampled data from offshore platforms have become available. The stochastic Lagrange model is a promising model, that agrees with observations and allows theoretical analysis of many statistical properties.

A thorough study of irregular, stochastic, Lagrange models was made by Gjøsund in [5], and the theory was further developed in [3] and [13].

In a series of recent papers, [1, 2, 6, 7, 9], Lindgren and Åberg have derived the exact statistical distributions of many wave characteristics, including steepness/slope, both for the space formulation, and for the time formulation.

In this paper we describe a modified Lagrange model with forced correlation between the vertical and horizontal components. The main message is that the statistical correlations between the vertical and horizontal movements uniquely determine the exact skewness and asymmetry distributions in the stochastic Lagrange model. We restrict the analysis to unidirectional 2D waves.
1 Stochastic Lagrange models

1.1 The Gauss-Lagrange model

A Lagrange wave is a stochastic version of a Miche wave, the depth dependent modification of a Gerstner wave [4, 11]. It describes the vertical and horizontal movements of individual water particle as functions of time \( t \) and original horizontal location \( u \), the reference co-ordinate. In the first order model, components with different frequencies and wave numbers act independently, and their effects are added. We consider here only particles on the free water surface.

The stochastic Lagrange model is obtained by letting the vertical and horizontal displacements be correlated random processes. The vertical process, which describes elevation above the still water level and is denoted \( W(t, u) \), is taken as a Gaussian process with mean zero, and so is the horizontal location, denoted \( X(t, u) \). Hence, a more complete name is Gauss-Lagrange model.

Expressed verbally, in the stochastic Lagrange model, a particle with still water location \((u, 0)\) has, at time \( t \), the stochastic coordinates \((X(t, u), W(t, u))\), and the height of the surface at location \( X(t, u) \) is therefore equal to \( W(t, u) \).

The Gauss-Lagrange model is completely defined by the covariance functions

\[
r_{ww}(t, u) = \text{Cov}(W(0, 0), W(t, u)) = \int_0^\infty \cos(\kappa u - \omega t) S(\omega) \, d\omega,
\]

and

\[
r_{xx}(t, u) = \text{Cov}(X(0, 0), X(t, u)),
\]

where \( S(\omega) \) is the orbital spectrum, and wave number \( \kappa > 0 \) and wave frequency \( \omega > 0 \) satisfy the dispersion relation,

\[
\omega^2 = g \kappa \tanh \kappa h,
\]

with water depth \( h \), and \( g \) denoting the gravitational constant.

1.2 The free stochastic Lagrange model

The free Gauss-Lagrange wave model is the stochastic version of the Miche waves. The horizontal displacement process \( X_M(t, u) \) is obtained as a linear filtration of the vertical process \( W(t, u) \) with amplitude and phase response function

\[
H_M(\omega) = \frac{i \cosh \kappa h}{\sinh \kappa h},
\]

where the subscript \( M \) stands for Miche filtration [3].

Expressed as stochastic Fourier integrals, the relation between the vertical and horizontal processes is

\[
W(t, u) = \int_{-\infty}^{\infty} e^{i(\kappa u - \omega t)} \, d\zeta(\omega),
\]

\[
X_M(t, u) = u + \int_{-\infty}^{\infty} e^{i(\kappa u - \omega t)} H_M(\omega) \, d\zeta(\omega),
\]

where the complex Gaussian process \( \zeta(\omega) \) has orthogonal increments such that

\[
E(\text{d}\zeta(\omega) \cdot \text{d}\zeta(\omega')) = \frac{1}{2} S(|\omega|), \quad \text{if } \omega = \omega'.
\]

The cross-covariance function between the vertical and horizontal process is

\[
r_{w\zeta}^M(t, u) = \int_0^\infty \cos(\kappa u - \omega t + \pi/2) \frac{\cosh \kappa h}{\sinh \kappa h} S(\omega) \, d\omega.
\]
1.3 The Lagrange model with linked components

In the free Lagrange model the water particles move according to the simplified hydrodynamic laws of motion, unaffected by outer forces, resulting in stochastically front-back symmetric waves. For wind driven waves this is unrealistic, and one would like to include some external influence in the interaction. A flexible approach is to replace (1) by a general response function,

$$H(\omega) = \rho(\omega) e^{i\theta(\omega)},$$

leading to the cross-covariance function of the form

$$r_{wx}(t, u) = \int_{-\infty}^{\infty} \cos(\kappa u - \omega t + \theta(\omega)) \rho(\omega) S(\omega) d\omega,$$

and a spectral representation with frequency dependent phase shift,

$$X(t, u) = u + \int_{-\infty}^{\infty} e^{i(\kappa u - \omega t + \theta(\omega))} \rho(\omega) d\zeta(\omega).$$

A physically motivated relation is obtained by letting the horizontal acceleration depend on the height, e.g. to take

$$\frac{\partial^2 X(t, u)}{\partial t^2} = \frac{\partial^2 X_M(t, u)}{\partial t^2} - \alpha W(t, u),$$

with $\alpha > 0$. With $G(\omega) = \frac{-\alpha}{(-\omega)^2}$, the response function will then be

$$H(\omega) = H_M(\omega) + G(\omega) = i \frac{\cosh \kappa h}{\sinh \kappa h} - \frac{\alpha}{(-\omega)^2} = \rho(\omega) e^{i\theta(\omega)}.$$

1.4 Time and space waves

The space wave is obtained as the parametric curve $u \mapsto (X(t_0, u), W(t_0, u))$, by keeping time $t = t_0$ fixed. A complication is that there may occur double points, where $X(t_0, u_1) = X(t_0, u_2)$ with $u_1 \neq u_2$ and $W(t_0, u_1) \neq W(t_0, u_2)$. The space wave is defined implicitly through the relation $L(t_0, X(t_0, u)) = W(t_0, u)$, and explicitly, if there is only one $u = X^{-1}(t_0, x)$ satisfying $X(t_0, u) = x$, as $L(t_0, x) = W(t_0, X^{-1}(t_0, x))$. The time wave is obtained as measurements of the free water level $L(t, x_0)$ at a fixed location in space with co-ordinate $x_0$, viz. as the curve $t \mapsto W(t, X^{-1}(t, x_0))$, provided that the inverse $X^{-1}(t, x_0) = \{u; X(t, u) = x_0\}$ is unique.

1.5 The example model

We will illustrate the theory on a model with Pierson-Moskowitz (PM) orbital spectrum, with spectral density

$$S(\omega) = \frac{5H_p^2}{\omega_p(\omega/\omega_p)^3} e^{-\frac{4}{3}(\omega/\omega_p)^{-4}}, \quad 0 < \omega < \omega_c,$$
with significant wave height $H_s = 4\sqrt{\text{Var}(W(t,u))} = 7$ m, peak frequency $\omega_p$, and peak period $T_p = 2\pi/\omega_p$. We assume a finite cut off frequency $\omega_c = 32/T_p$ to obtain finite spectral moments. The steepness parameter, $H_s/T_p^2$, is important for the front-back asymmetry, and we illustrate the distributions for very steep, steep, and moderately steep waves with $T_p = 12, 14, 16$ s. We do the calculations for different depths, $h = 8, 32, 64, \infty$ m and three different degrees of linkage, $\alpha = 0, 0.4, 0.8$, where $\alpha = 0$ means “no linkage”.

### 2 Wave characteristics

#### 2.1 Observable characteristics

There are many wave characteristic distributions that are of importance in ocean engineering practice, and they can all be derived exactly from the correlation properties. In different applications one can identify many different quantities related to the wave profiles. Some are described by their statistical distribution when waves are sampled at a constant sampling rate. Others are coupled to level crossings and the wave profile when the wave reaches some specified level. We list five variables of special interest that we analyse in the examples:

(A) **Asynchronous slopes in space**: This is the distribution of the space slope $L_x(t_0, x)$ observed at equidistant sampling of the space wave.

(AT) **Asynchronous slopes in time**: This is the distribution of the time slope $L_t(t, x_0)$ observed at constant rate sampling of the time wave.

(SS) **Slope in space at level crossings in space**: This is the distribution of the space slope $L_x(t_0, x)$ observed only at the up- or downcrossings of a fixed level $v$ by the space wave $L(t_0, x)$, (synchron sampling in space).

(TT) **Slope in time at level crossings in time**: This is the distribution of the time slope $L_t(t, x_0)$ observed only at the up- or downcrossings of a fixed level $v$ by the time wave $L(t, x_0)$, (synchron sampling in time).

(ST) **Slope in space at level crossings in time**: This is the distribution of the space slope $L_u(t, x_0)$ observed at the instances when the time wave reaches level $v$. Note that this is the slope of the moving wave front that may hit a deck of an offshore construction.

The observable statistical distributions in the different cases are defined as the asymptotic, long run, empirical distributions.

#### 2.2 Model characteristics

To find the theoretical statistical distribution of the observable characteristics in cases (US)-(ST) we need to express the corresponding quantities in terms of the vertical and horizontal processes, $W(t,u)$ and $X(t,u)$. We derive the model variables in terms of the partial time and space derivatives, which we denote as $X_t(t,u) = \partial X(t,u)/\partial t$, $X_u(t,u) = \partial X(t,u)/\partial u$, $W_t(t,u) = \partial W(t,u)/\partial t$, $W_u(t,u) = \partial W(t,u)/\partial u$, etc.
Consider first the time wave. The Lagrange time wave satisfies the relation \( L(t, X(t, u)) = W(t, u) \). By differentiating with respect to \( t \), we obtain
\[
\frac{\partial L(t, X(t, u))}{\partial t} = W_t(t, u) = L_t(t, X(t, u)) + L_u(t, X(t, u)) X_t(t, u).
\]
By differentiating with respect to \( u \), we obtain, \( W_u(t, u) = L_u(t, X(t, u)) X_u(t, u) \), giving the fundamental definition of the time wave slope at location \( X(t, u) \),
\[
L_t(t, X(t, u)) = W_t(t, u) - W_u(t, u) \frac{X_t(t, u)}{X_u(t, u)}.
\]

Equation (2) is a mathematical identity, and if \( X^{-1}(t, x_0) \) is uniquely defined it also gives the unique slope of the Lagrange time wave \( L(t, x_0) \) at location \( x_0 \). If there are multiple \( u \)-values such that \( X(t, u) = x_0 \), we define \( L_t(t, x_0) \) by (2) for each of these \( u \)-values.

We also need the slope of the space wave observed at time \( t_0 \). The space wave is implicitly defined by \( L(t_0, X(t_0, u)) = W(t_0, u) \), or explicitly, if there is only one \( u = X^{-1}(t_0, x) \) satisfying \( X(t_0, u) = x \), by \( L(t_0, x) = W(t_0, X^{-1}(t_0, x)) \). For each of the solutions, the slope is defined by
\[
L_x(t_0, x) = \frac{W_u(t_0, u)}{X_u(t_0, u)}.
\]

3 Asyncronous slopes, exact results for (AS) and (AT)

The asynchronous slopes are obtained by asynchronous sampling, for example equidistant in time or in space, and their model representatives are defined by (2) and (3). To find the slope distributions at a location \( x_0 \) at time \( t_0 \) one has to identify the reference coordinate for the particle that occupies position \( x_0 \) at time \( t_0 \), i.e. to find the solution(s) to the equation \( X(t_0, u) = x_0 \). Due to the stationarity in time and space we can take \( t_0 = 0 \), \( x_0 = 0 \).

Now, \( X(0, u) \) is a Gaussian process with parameter \( u \), with non-constant mean \( u \), and stationary covariance function \( r^{xx}(0, u) = \text{Cov}(X(0, 0), X(t, u)) \). The slope distributions in time or space are equal to the distribution of the representations (2) and (3), respectively, under the condition that \( u \) is a point of crossing of the level 0 by the process \( X(0, u) \). To reduce the effect of multiple solutions as much as possible, we consider only upcrossings.

The number of solutions to \( X(0, u) = 0 \) is random, with a small probability of there being more than one, and so are the reference coordinates of the solutions. To formulate the long run distribution, we define the counters \( N^+ = \# \{ u; X(0, u) = 0, \text{upcrossing} \} \), and
\[
N^+(a, b) = \# \left\{ u; X(0, u) = 0, \text{upcrossing}, a < \frac{W_u(0, u)}{X_u(0, u)} \leq b \right\},
\]
\[
N^+(a, b) = \# \left\{ u; X(0, u) = 0, \text{upcrossing}, a < \frac{W_u(0, u)}{X_u(0, u)} \leq b \right\}.
\]
The distribution functions of the asynchronous slopes in time and space, are then
\[ F(L_T \leq x) = \frac{E(N^{T}(-\infty, x))}{E(N^{+})} \quad \text{and} \quad F(L_S \leq x) = \frac{E(N^{S}(-\infty, x))}{E(N^{+})}. \]

Define the indicator functions
\[ I^{(T)}(a, b) = I\left\{ a < W_t(0, u) - W_u(0, u) \frac{X_t(0, u)}{X_u(0, u)} \leq b \right\}, \quad (4) \]
\[ I^{(S)}(a, b) = I\left\{ a < W_u(0, u) \leq b \right\}, \quad (5) \]
and let \( p_u(0, z) \) be the density of \( U(0, u) = (X(0, u), X_u(0, u)) \) at \((0, z)\).

**Theorem 1** ([8]). The observable distributions of the asynchronous time and space slopes in the Gauss-Lagrange model are given by,
\[ F(L_S \leq y) = \frac{1}{E(N^{+})} \int_{u=-\infty}^{\infty} \int_{z=0}^{\infty} z \, p_u(0, z) \, E\left( I^{S}(-\infty, y) \mid U(0, u) = (0, z) \right) \, dz \, du, \quad (6) \]
\[ F(L_T \leq y) = \frac{1}{E(N^{+})} \int_{u=-\infty}^{\infty} \int_{z=0}^{\infty} z \, p_u(0, z) \, E\left( I^{T}(-\infty, y) \mid U(0, u) = (0, z) \right) \, dz \, du, \quad (7) \]
with \( E(N^{+}) = \int_{u=-\infty}^{\infty} \int_{z=0}^{\infty} z \, p_u(0, z) \, dz \, du. \) The expectations in (6)–(7) are easily found by conditional simulation of the three-dimensional Gaussian vector \((W_t(t, u), W_u(t, u), X_t(t, u))\) given that \( U(0, u) = (0, z)\).

**Example 1.** We use the Pierson-Moskowitz (PM) orbital spectrum described in Section 1.5 and simulate the vertical and horizontal components. From these we construct space and time waves and observe the slopes. We also compute, by Monte Carlo simulation of the expectations, the exact cumulative distribution functions.

We simulated 50 replicates of space and time waves, with length 2048 m and 2048 s, respectively. Table 1 shows the estimated skewness (with standard error) for three water depths, and three degrees of linking dependence. The skewness (A) and kurtosis excess (B) in the observed slopes are used as an estimate of the theoretical skewness of the slopes and deviation from normality, (with subscript \( i = S, T \) indicating space or time waves), \( A_i = \frac{E(L_{slope}^3)}{\text{Var}(L_{slope})^{3/2}}, \quad B_i = \frac{E(L_{slope}^4)}{\text{Var}(L_{slope})^2} - 3. \)

Figure 1 shows cumulative space slope distributions for the nine combinations of depth and linking. As seen the slope distribution deviates significantly from normality, even in in the symmetric case with infinite water depth. For time waves, the skewness is not as extreme as for space waves (not shown here).
Table 1. Estimated skewness and kurtosis excess with standard errors of asynchronous slope distributions, PM orbital spectrum with $H_s = 7m$, $T_p = 11s$.

<table>
<thead>
<tr>
<th>Depth</th>
<th>$\alpha = 0$</th>
<th>$\alpha = 0.4$</th>
<th>$\alpha = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = \infty$</td>
<td>$A_S = 0.0 (0.02)$</td>
<td>$-0.6 (0.03)$</td>
<td>$-0.6 (0.03)$</td>
</tr>
<tr>
<td></td>
<td>$A_T = 0.0 (0.01)$</td>
<td>$0.5 (0.01)$</td>
<td>$0.9 (0.02)$</td>
</tr>
<tr>
<td></td>
<td>$B_S = 0.2 (0.03)$</td>
<td>$0.4 (0.05)$</td>
<td>$1.2 (0.14)$</td>
</tr>
<tr>
<td></td>
<td>$B_T = 0.1 (0.02)$</td>
<td>$0.4 (0.05)$</td>
<td>$1.4 (0.07)$</td>
</tr>
<tr>
<td>$h = 64$</td>
<td>$A_S = 0.0 (0.02)$</td>
<td>$-0.3 (0.02)$</td>
<td>$-0.7 (0.03)$</td>
</tr>
<tr>
<td></td>
<td>$A_T = 0.0 (0.01)$</td>
<td>$0.5 (0.01)$</td>
<td>$0.9 (0.02)$</td>
</tr>
<tr>
<td></td>
<td>$B_S = 0.1 (0.04)$</td>
<td>$0.3 (0.05)$</td>
<td>$1.3 (0.16)$</td>
</tr>
<tr>
<td></td>
<td>$B_T = 0.1 (0.02)$</td>
<td>$0.5 (0.03)$</td>
<td>$1.4 (0.09)$</td>
</tr>
<tr>
<td>$h = 8$</td>
<td>$A_S = 0.0 (0.02)$</td>
<td>$-1.1 (0.08)$</td>
<td>$-2.6 (0.12)$</td>
</tr>
<tr>
<td></td>
<td>$A_T = 0.0 (0.03)$</td>
<td>$0.8 (0.05)$</td>
<td>$1.5 (0.08)$</td>
</tr>
<tr>
<td></td>
<td>$B_S = 0.8 (0.08)$</td>
<td>$2.2 (0.66)$</td>
<td>$7.2 (1.41)$</td>
</tr>
<tr>
<td></td>
<td>$B_T = 1.1 (0.08)$</td>
<td>$2.3 (0.26)$</td>
<td>$5.0 (0.29)$</td>
</tr>
</tbody>
</table>

Fig. 1. Cumulative distribution functions (CDF) on normal probability paper for asynchronous slopes in Lagrange space waves. Crosses = simulated data, solid line = theoretical CDF.

4 Asymmetry in space waves, case (SS)

The distribution of the space wave slope at a crossing of the level $v$ is the distribution of the ratio (3), conditioned on the event that $u$ is a $v$-crossing point in the vertical Gaussian process $W(t_0, u)$. Now, it is wellknown that the derivative of a Gaussian process, observed only at crossings of a fixed level, has a two-sided Rayleigh distribution. Observed at upcrossings or downcrossings, the derivatives have a positive or negative Rayleigh distribution, respectively.
In the following theorem, $R$ and $U$ are two independent random variables, with densities, respectively, $f_R(r) = \frac{|r|}{\sqrt{2\pi}} e^{-r^2/2}$, and $f_U(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$, i.e. $R$ has a two-sided Rayleigh distribution and $U$ is standard normal. The notation $X \overset{d}{=} Y$ means that the random variables $X$ and $Y$ have the same distribution.

**Theorem 2 ([1, 9]).** The distribution of the space derivatives in (3), under the condition that $u_k$ is a $v$-crossing point in $W(t_0, u)$, can be expressed as

\[
W_u(t_0, u_k) \overset{d}{=} R \sqrt{r_{uu}} + \sqrt{r_{uu} - (r_{uu} u_0)^2 - (r_{uu} u)^2},
\]

\[
X_u(t_0, u_k) \overset{d}{=} 1 + V \frac{r_{uu}}{r_{uu} u_0} + R \frac{r_{uu} x}{r_{uu} u_0} + U \sqrt{r_{uu} - (r_{uu} u_0)^2 - (r_{uu} u)^2},
\]

and hence the slope of the Lagrange space wave at a $v$-crossing has the representation

\[
L_x(t_0, x_k) \overset{d}{=} \frac{R \sqrt{r_{uu}}}{1 + V \frac{r_{uu}}{r_{uu} u_0} + R \frac{r_{uu} x}{r_{uu} u_0} + U \sqrt{r_{uu} - (r_{uu} u_0)^2 - (r_{uu} u)^2}},
\]

(8)

The distributions at an upcrossing or at a downcrossing are obtained by replacing the two-sided Rayleigh variable by a one-sided, positive and negative, respectively. An explicit formula for the probability density can be found in [9].

As seen in the denominator in (8), the front-back asymmetry depends on the covariance

\[
r_{uu} = \int_0^\infty \kappa^2 \cos(\theta(\omega)) S(\omega) d\omega,
\]

between the spatial derivatives of the vertical and horizontal processes. If it is zero the slope distribution at an upcrossing is just the mirror of that at a downcrossing. If non-zero the Rayleigh variable in the nominator also influences the denominator and makes the slope distribution asymmetric. In the free Lagrange model the phase shift is $90^\circ$ and the covariance is 0.

**Example 2:** Figure 2 shows how the the cumulative distribution functions (CDF) for the upcrossing and downcrossing slopes in Lagrange space waves with $\alpha = 0.4$, depend on the crossed level, and on the water depth and wave steepness.

5 **Asymmetry in time waves, cases (TT), (ST)**

The time wave level crossings at position $x_0$ are more complicated than the space wave crossings, since one has to follow the vertical and horizontal variations of the random particle that happens to be located at $x_0$ as time changes. Thus, a crossing of the level $v$ occurs at time $t_k$ if there is a particle with reference coordinate, which we denote by $u_k$, such that $W(t_k, u_k) = v$ and $X(t_k, u_k) = x_0$. To solve problems (TT), (ST), we have to find the conditional distribution of
the time slope, defined by (2), and space slope, defined by (3), conditioned on the crossing event, just defined.

By a remarkable generalization of Rice’s formula for the number of level crossings, Mercardier, [10], has given the tool for how to find conditional distributions like the ones we seek. To formulate the theorem, we define

\[ D = \left| W_t(0, u)X_u(0, u) - W_u(0, u)X_t(0, u) \right|, \]

and write \( g_u(v, x) \) for the density of \( V(0, u) = (W(0, u), X(0, u)) \), evaluated at \((v, x)\).

**Theorem 3** ([1, 7]). (a) The distribution function for slopes at upcrossings of the level \( v \) in the Lagrange time wave, is given by,

\[ F_T^T(v) = \frac{1}{E(N^+)} \int_{-\infty}^{\infty} g_u^{TT+}(u) q_u(v, x_0) \, du, \]

where

\[ g_u^{TT+}(u) = E(D \times I(T)(0, y) \mid V(0, u) = (v, x_0)), \]

and \( E(N^+) = \int_{-\infty}^{\infty} g_u^{TT+}(u) g_w(0, u)X(0, u)(v, x_0) \, du. \)

(b) The cumulative distribution of slopes at downcrossings is obtained by replacing the indicator function \( I(T)(0, y) \) by \( I(T)(-y, 0) \), properly adjusting the \( \leq \) sign. The distribution for space slope, \((ST)\), at time crossings is found by replacing (4) by (5) in (9).
Fig. 3. Cumulative distribution functions for time wave slopes (absolute values) at time wave crossings of different levels. Slope CDF at upcrossings (solid lines) and at downcrossings (dash-dotted lines). Levels \( v = [-1, 0, 1, 2, 3] \times \sigma, 4\sigma = H_s \). Largest absolute values correspond to highest level. Orbital spectrum is PM with \( T_p = 12s \).

Example 3: Figures 3 and 4 illustrate cases (TT) and (ST), i.e. distributions of slopes in time and space, respectively, observed at up- and downcrossings of different levels. The case (ST), space slopes at time wave crossings, has drawn some interest in the study of rogue waves. It is related to the problem of relation between time wave formulation and the properties of observed wavelength troughs in space, in particular the question of “a hole in the sea” ahead of extreme waves. A common technique to approach this problem is the Fourier snapshot method, described in [12]. The space-time analysis presented in this paper might be an adequate method for genuinely asymmetric waves.

6 Concluding remark

The main message is that the full covariance structure of horizontal and vertical water particle velocities can be the basis for a statistical analysis of crest-front asymmetry of real ocean waves. Besides observational data also data from Monte Carlo experiments of fully non-linear waves models should be of interest.

References

Fig. 4. Cumulative distribution functions for space wave slopes at time wave crossings of different levels. Slope CDF at upcrossings (solid lines) and at downcrossings (dash-dotted lines). Levels $v$: $[-1, 0, 1, 2, 3] \times \sigma$, $4\sigma = H_s$. Most extreme values correspond to highest level. Orbital spectrum is PM with $T_p = 12s$.